MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

VOL. LXXVII. APRIL 13, 1917. No. 6

Major P. A. MACMAHON, D.Sc., Sc.D., LL.D., F.R.S., PRESIDENT, in the Chair.

James Bootherstone, 38 St. Annes Road, Chorlton-cum-Hardy, Manchester,

was balloted for and duly elected a Fellow of the Society.

The following were proposed by the Council as Associates of the Society:—

Professor C. G. Abbot, Director of the Astrophysical Observatory of the Smithsonian Institution, Washington, D.C., U.S.A.;

Professor Giovanni Celoria, Director, Reale Osservatorio di Brera, Milan, Italy;

M. Pierre Henri Puiseux, Observatoire, Paris; and

Professor Charles Edward St. John, F.R.A.S., Solar Observatory, Mount Wilson, California, U.S.A.

Fifty-one presents were announced as having been received since the last meeting, including, amongst others:—

David Brunt, The Combination of Observations, presented by the author.

Rome, Specola Vaticana, Aggiunte al Catalogo dell' Atlas Stellarum Variabilium di J. G. Hagen, presented by the Observatory.

A Determination of the Constant of Aberration. By S. S. Hough, M.A., F.R.S., H.M. Astronomer at the Cape. (Plates 4, 5.)

During the year 1911 the new reversible transit-circle of this Observatory was devoted exclusively to the observation of a series of close circumpolar stars. A discussion of the observations with a view to the determination of the most accurate positions of the stars is contained in *Cape Annals*, vol. xii. part 3, and it is there indicated how the observations may be utilised to derive a determination of the constant of aberration with no inconsiderable weight. As this determination did not form part of the programme originally intended when the observations were made, no special provision was made to secure observations under the most favourable conditions for yielding large aberration factors. The object of the present note is primarily to correct inaccuracies in the provisional procedure adopted in the *Cape Annals*, and to derive the value of the aberration constant by a more rigorous discussion of the material; and further, to consider how the programme of observations might best be modified with a view to repetition, so as to throw as much weight as possible into the determination of this quantity.

For the purposes of the former discussion the observations were divided into twenty-six groups, each contained within a limited period not exceeding about a fortnight in duration, during which the azimuth of the meridian marks and the instrumental latitude were regarded as sensibly constant. This same grouping has been adhered to in the present work.

The first step was to evaluate for each star in each group mean values of the aberration factors as involved in the computation of the "star correction."

Denoting by Δk the correction to the tabular value of the aberration constant ($20''\cdot47$) as used in the *Nautical Almanac*, the additional corrections to be applied to the observations to reduce from apparent to mean place, consequent on this change in the value of the constant of aberration (k), are given by the formulæ

$$\Delta\alpha \cos\delta = -h \sin(\mathrm{H}+\alpha)\Delta k/k,$$
$$\Delta\delta \quad\quad = -h \sin\delta \cos(\mathrm{H}+\alpha)\Delta k/k.$$

In estimating the coefficients, $\sin\delta$ was throughout taken as -1, and the values of H, h/k for the separate groups were derived from the following rough table (Table I.).

A rough mean value of α for each separate star was adopted.

TABLE I.

	H.		h/k.		H.		h/k
	h	m			h	m	
Jan. 16–26	22	0	·98	July 4–19	10	48	·99
Jan. 28–Feb. 8	21	0	·96	July 20–Aug. 8	9	44	·97
Feb. 16–23	20	0	·94	Aug. 12–26	8	24	·95
Feb. 24–Mar. 3	19	20	·92	Aug. 28–Sept. 2	7	40	·93
Mar. 13–20	18	16	·92	Sept. 4–11	7	0	·92
Mar. 21–30	17	40	·92	Sept. 12–19	6	40	·92
Apr. 1–8	17	0	·93	Sept. 28–Oct. 7	5	20	·92
Apr. 10–24	16	0	·94	Oct. 9–20	4	30	·92
Apr. 28–May 12	15	0	·96	Oct. 25–28	3	40	·94
May 14–27	14	0	·98	Oct. 30–Nov. 6	3	10	·95
May 30–June 9	13	0	·99	Nov. 7–12	2	40	·96
June 10–21	12	20	1·00	Nov. 13–19	2	12	·97
June 25–July 3	11	30	1·00	Nov. 20–29	1	44	·98

Retaining Δk as an algebraic quantity, associated with numerical coefficients derived from this table, means of these coefficients in the different group combinations were taken out, following exactly the same procedure as that used in the original discussion, in which the polar corrections in R.A. and latitude corrections in declination, and subsequently the star places, were derived. For instance, the expressions for polar deviations in R.A. are given in the following table.

TABLE II.

Polar Deviation in Right Ascension, including Terms due to Aberration Correction.

	Δp.			Δp.	
	s			s	
Jan. 16–26	−0·030	−1·44 Δk	July 4–19	+0·014	·00 Δk
Jan. 28–Feb. 8	−0·014	−1·27 Δk	July 20–Aug. 8	+0·046	·00 Δk
Feb. 16–23	−0·022	−1·16 Δk	Aug. 12–26	+0·029	+0·24 Δk
Feb. 24–Mar. 3	+0·012	−1·05 Δk	Aug. 28–Sept. 2	+0·019	+0·30 Δk
Mar. 13–20	+0·013	−0·85 Δk	Sept. 4–11	+0·018	−0·46 Δk
Mar. 22–30	−0·061	−0·72 Δk	Sept. 12–19	+0·021	−0·55 Δk
Apr. 1–8	+0·025	−0·60 Δk	Sept. 28–Oct. 7	+0·011	−0·88 Δk
Apr. 12–24	−0·033	−0·46 Δk	Oct. 9–20	+0·030	−0·83 Δk
Apr. 28–May 12	+0·010	0·00 Δk	Oct. 25–29	+0·015	−0·87 Δk
May 14–27	−0·013	0·00 Δk	Oct. 30–Nov. 6	+0·032	−0·88 Δk
May 30–June 9	+0·040	0·00 Δk	Nov. 7–12	−0·001	−0·93 Δk
June 10–21	+0·005	0·00 Δk	Nov. 13–19	+0·032	−1·28 Δk
June 25–July 3	+0·031	0·00 Δk	Nov. 19–29	+0·007	−1·18 Δk

The constant parts in these expressions correspond with those of the original discussion, the terms involving Δk being now added. The coefficients of Δk during the period April 28–August 8 are throughout zero, as the determinations of azimuth therein involved are based solely on observations of the same stars at adjacent upper and lower culminations.

At these epochs the effects of an error in the adopted tabular aberration constant will be thrown entirely into the derived mean right ascensions of the stars. In the case of the stars observed at double transit it will be always associated with a small coefficient, as the observations are made near the times of sunrise and sunset, when the value of $H + \alpha$ is near zero or twelve hours. This, however, will not be the case for other stars observed in the same watches whose right ascensions sometimes differ by as much as four hours from the former, giving a factor of Δk, in the expression for $\Delta\alpha \cos\delta$, amounting in the extreme to about $\pm 0^{s}.87$.

The coefficients of Δk appropriate to the other epochs thus involve the mean differences between the factors for each star at the epoch at which the value of Δp is sought and that at which the provisional determinations of right ascension were made. In the former discussion the second part of the factor was treated (erroneously) as zero, both in the right ascensions and in the analogous treatment of the declinations.

The revised expressions for the latitude corrections, including the aberration terms, are as follows :—

TABLE III.

Latitude Corrections, including Aberration Terms.

	$\Delta\phi$.			$\Delta\phi$.	
Jan. 16–26	$+0''.25$	$-1''.25\ \Delta k$	July 4–19	$-0''.16$	$0''.00\ \Delta k$
Jan. 28–Feb. 8	$-0''.47$	$-1''.27\ \Delta k$	July 20–Aug. 8	$-0''.04$	$0''.00\ \Delta k$
Feb. 16–23	$-0''.13$	$-0''.43\ \Delta k$	Aug. 12–26	$-0''.06$	$-0''.21\ \Delta k$
Feb. 24–Mar. 3	$-0''.29$	$-0''.23\ \Delta k$	Aug. 28–Sept. 2	$-0''.18$	$-0''.39\ \Delta k$
Mar. 13–20	$-0''.33$	$+0''.42\ \Delta k$	Sept. 4–11	$+0''.04$	$-0''.97\ \Delta k$
Mar. 22–30	$-0''.48$	$+0''.40\ \Delta k$	Sept. 12–19	$-0''.05$	$-1''.25\ \Delta k$
Apr. 1–8	$-0''.66$	$+0''.15\ \Delta k$	Sept. 28–Oct. 7	$+0''.12$	$-1''.24\ \Delta k$
Apr. 12–24	$-0''.56$	$+0''.20\ \Delta k$	Oct. 9–20	$+0''.05$	$-1''.40\ \Delta k$
Apr. 28–May 12	$-0''.43$	$0''.00\ \Delta k$	Oct. 25–28	$+0''.07$	$-1''.44\ \Delta k$
May 14–27	$-0''.28$	$0''.00\ \Delta k$	Oct. 30–Nov. 6	$-0''.01$	$-1''.47\ \Delta k$
May 30–June 9	$-0''.43$	$0''.00\ \Delta k$	Nov. 7–12	$+0''.29$	$-1''.48\ \Delta k$
June 10–21	$-0''.49$	$0''.00\ \Delta k$	Nov. 13–19	$-0''.07$	$-1''.02\ \Delta k$
June 25–July 3	$-0''.34$	$0''.00\ \Delta k$	Nov. 20–29	$+0''.22$	$-1''.10\ \Delta k$

In like manner the appropriate factors of Δk were calculated for correcting the derived right ascensions and declinations of the stars as contained in Table XXV. of the former paper. The following expressions for $\Delta\alpha \cos\delta$, $\Delta\delta$ were thus derived.

TABLE IV.

Aberration Corrections for Separate Stars.

	$\Delta\alpha \cos\delta$.	$\Delta\delta$.		$\Delta\alpha \cos\delta$.	$\Delta\delta$.
Lacaille 9745	$+0^s\cdot1\ \Delta k$,	$-1''\cdot0\ \Delta k$	Lacaille 4991	$-0^s\cdot1\ \Delta k$,	$+0''\cdot9\ \Delta k$
Lacaille 23	$+0^s\cdot1\ \Delta k$,	$-1''\cdot0\ \Delta k$	Lacaille 5096	$-0^s\cdot1\ \Delta k$,	$+1''\cdot0\ \Delta k$
o Octantis	$+0^s\cdot1\ \Delta k$,	$-0''\cdot9\ \Delta k$	Lacaille 5107	$-0^s\cdot1\ \Delta k$,	$+0''\cdot9\ \Delta k$
β Hydri	$+0^s\cdot2\ \Delta k$,	$-0''\cdot9\ \Delta k$	Lacaille 5235	$-0^s\cdot2\ \Delta k$,	$+0''\cdot8\ \Delta k$
Lacaille 228	$+0^s\cdot2\ \Delta k$,	$-0''\cdot9\ \Delta k$	ι Octantis	$-0^s\cdot2\ \Delta k$,	$+0''\cdot9\ \Delta k$
Lacaille 248	$+0^s\cdot1\ \Delta k$,	$-0''\cdot9\ \Delta k$	Lacaille 5325	$-0^s\cdot2\ \Delta k$,	$+0''\cdot9\ \Delta k$
Gilliss P.Z. 939	$+0^s\cdot1\ \Delta k$,	$-1''\cdot0\ \Delta k$	Lacaille 5452	$-0^s\cdot2\ \Delta k$,	$+1''\cdot0\ \Delta k$
Gilliss P.Z. 1219	$+0^s\cdot1\ \Delta k$,	$-1''\cdot0\ \Delta k$	Lacaille 5444	$-0^s\cdot2\ \Delta k$,	$+1''\cdot0\ \Delta k$
Lacaille 634	$+0^s\cdot3\ \Delta k$,	$-1''\cdot0\ \Delta k$	κ Octantis	$-0^s\cdot2\ \Delta k$,	$+0''\cdot9\ \Delta k$
Lacaille 760	$+0^s\cdot2\ \Delta k$,	$-0''\cdot9\ \Delta k$	C.G.A. 18500	$-0^s\cdot2\ \Delta k$,	$+1''\cdot0\ \Delta k$
Lacaille 764	$+0^s\cdot2\ \Delta k$,	$-1''\cdot0\ \Delta k$	δ Octantis	$-0^s\cdot2\ \Delta k$,	$+0''\cdot9\ \Delta k$
Lacaille 1884	$+0^s\cdot3\ \Delta k$,	$-0''\cdot9\ \Delta k$	Brisbane 4614	$-0^s\cdot3\ \Delta k$,	$+0''\cdot9\ \Delta k$
Lacaille 1029	$+0^s\cdot3\ \Delta k$,	$-0''\cdot9\ \Delta k$	Gilliss P.Z. 10018	$-0^s\cdot3\ \Delta k$,	$+0''\cdot8\ \Delta k$
Lacaille 1076	$+0^s\cdot2\ \Delta k$,	$-0''\cdot9\ \Delta k$	Lacaille 5882	$-0^s\cdot4\ \Delta k$,	$+0''\cdot9\ \Delta k$
Lacaille 1203	$+0^s\cdot4\ \Delta k$,	$-0''\cdot9\ \Delta k$	z Octantis	$-0^s\cdot3\ \Delta k$,	$+1''\cdot0\ \Delta k$
Lacaille 1848	$+0^s\cdot3\ \Delta k$,	$-0''\cdot9\ \Delta k$	ρ Octantis	$-0^s\cdot3\ \Delta k$,	$+0''\cdot9\ \Delta k$
Lacaille 1414	$+0^s\cdot5\ \Delta k$,	$-0''\cdot9\ \Delta k$	Lacaille 6441	$-0^s\cdot5\ \Delta k$,	$+0''\cdot7\ \Delta k$
Lacaille 1839	$+0^s\cdot6\ \Delta k$,	$-0''\cdot7\ \Delta k$	Lacaille 6545	$-0^s\cdot5\ \Delta k$,	$+0''\cdot8\ \Delta k$
Lacaille 1707	$+0^s\cdot5\ \Delta k$,	$-0''\cdot7\ \Delta k$	Gilliss P.Z. 11448	$-0^s\cdot5\ \Delta k$,	$+0''\cdot8\ \Delta k$
Cape 1880, 2449	$+0^s\cdot7\ \Delta k$,	$-0''\cdot7\ \Delta k$	C.G.A. 23027	$-0^s\cdot6\ \Delta k$,	$+0''\cdot7\ \Delta k$
Lacaille 2296	$+0^s\cdot5\ \Delta k$,	$-0''\cdot5\ \Delta k$	Lacaille 7078	$-0^s\cdot6\ \Delta k$,	$+0''\cdot6\ \Delta k$
Cape 1880, 2901	$+0^s\cdot5\ \Delta k$,	$-0''\cdot4\ \Delta k$	χ Octantis	$-0^s\cdot5\ \Delta k$,	$+0''\cdot4\ \Delta k$
Lacaille 2512	$+0^s\cdot5\ \Delta k$,	$-0''\cdot4\ \Delta k$	Brisbane 6229	$-0^s\cdot5\ \Delta k$,	$+0''\cdot1\ \Delta k$
Lacaille 3274	$+0^s\cdot5\ \Delta k$,	$+0''\cdot4\ \Delta k$	Lacaille 7442	$-0^s\cdot4\ \Delta k$,	$+0''\cdot1\ \Delta k$
Gilliss P.Z. 5752	$+0^s\cdot2\ \Delta k$,	$0''\cdot0\ \Delta k$	σ Octantis	$-0^s\cdot4\ \Delta k$,	$-0''\cdot4\ \Delta k$
A Octantis	$+0^s\cdot3\ \Delta k$,	$+0''\cdot4\ \Delta k$	Lacaille 7751	$-0^s\cdot4\ \Delta k$,	$-0''\cdot1\ \Delta k$
Brisbane 2007	$+0^s\cdot2\ \Delta k$,	$+0''\cdot6\ \Delta k$	Gilliss P.Z. 13504	$-0^s\cdot2\ \Delta k$,	$-0''\cdot2\ \Delta k$
θ Chamæleontis	$+0^s\cdot2\ \Delta k$,	$+0''\cdot8\ \Delta k$	Lacaille 8094	$-0^s\cdot3\ \Delta k$,	$-0''\cdot4\ \Delta k$
Lacaille 3759	$+0^s\cdot2\ \Delta k$,	$+0''\cdot8\ \Delta k$	Lacaille 8257	$-0^s\cdot2\ \Delta k$,	$-0''\cdot6\ \Delta k$
Gilliss P.Z. 6020	$+0^s\cdot2\ \Delta k$,	$+0''\cdot4\ \Delta k$	C.G.A. 28663	$0^s\cdot0\ \Delta k$,	$-0''\cdot8\ \Delta k$
ζ Octantis	$0^s\cdot0\ \Delta k$,	$+0''\cdot9\ \Delta k$	C.G.A. 28714	$0^s\cdot0\ \Delta k$,	$-0''\cdot8\ \Delta k$
Lacaille 4342	$-0^s\cdot1\ \Delta k$,	$+0''\cdot9\ \Delta k$	Lacaille 8551	$0^s\cdot0\ \Delta k$,	$-0''\cdot8\ \Delta k$
C.G.A. 14444	$-0^s\cdot1\ \Delta k$,	$+9''\cdot9\ \Delta k$	Lacaille 8751	$0^s\cdot0\ \Delta k$,	$-0''\cdot9\ \Delta k$
C.G.A. 14481	$-0^s\cdot1\ \Delta k$,	$+0''\cdot9\ \Delta k$	Lacaille 8720	$0^s\cdot0\ \Delta k$,	$-0''\cdot8\ \Delta k$
Lacaille 4510	$-0^s\cdot1\ \Delta k$,	$+0''\cdot9\ \Delta k$	B Octantis	$0^s\cdot0\ \Delta k$,	$-0''\cdot9\ \Delta k$
δ^2 Chamæleontis	$-0^s\cdot1\ \Delta k$,	$+0\ 8\ \Delta k$	Lacaille 8738	$0^s\cdot0\ \Delta k$,	$-0''\cdot9\ \Delta k$
Lacaille 4578	$-0^s\cdot1\ \Delta k$,	$+0''\cdot9\ \Delta k$	υ (C.) Octantis	$+0^s\cdot1\ \Delta k$,	$-0''\cdot9\ \Delta k$
η Octantis	$-0^s\cdot1\ \Delta k$,	$+0''\cdot9\ \Delta k$	β Octantis	$+0^s\cdot2\ \Delta k$,	$-0''\cdot9\ \Delta k$
Lacaille 4708	$-0^s\cdot1\ \Delta k$,	$+0''\cdot8\ \Delta k$	τ Octantis	$+0^s\cdot1\ \Delta k$,	$-0''\cdot9\ \Delta k$
Lacaille 4731	$-0^s\cdot1\ \Delta k$,	$+0''\cdot9\ \Delta k$	Lacaille 9464	$+0^s\cdot1\ \Delta k$,	$-0''\cdot9\ \Delta k$
Gilliss P.Z. 7980	$-0^s\cdot2\ \Delta k$,	$+0''\cdot9\ \Delta k$	Lacaille 9563	$+0^s\cdot1\ \Delta k$,	$-0''\cdot9\ \Delta k$
C.G.A. 15761	$0^s\cdot0\ \Delta k$,	$+0''\cdot9\ \Delta k$	Lacaille 9596	$+0^s\cdot1\ \Delta k$,	$-0''\cdot9\ \Delta k$
Lacaille 4865	$-0^s\cdot1\ \Delta k$,	$+0''\cdot9\ \Delta k$			

In this table the order of arrangement for the different stars is that of R.A. It will be seen that with the actual arrangement of the observations the effect of an error in the adopted aberration constant would be to produce a pronounced periodic error in the resulting declinations, and a similar periodic error, but of a less pronounced character and in complementary phase, in the right ascensions.

In the discussion given later the aim has been to re-arrange the observations through the year in such a manner as to throw these periodic errors into higher relief, and at the same time provide means for strengthening their determination.

Still retaining Δk in algebraic form, residuals were now derived by comparing the separate observations (combined in groups, but uncorrected for latitude and azimuth) with the final results derived from the combination of all groups. For example, taking the period Sept. 4–11, the right ascension observations yielded the following residuals.

TABLE V.

Residuals (*O–C*). *September* 4–11.

Above-pole Observations.	s		No.	Below-pole Observations.	s		No.
χ Octantis	+0·023	+0·2 Δk	4	Lacaille 2296	−0·049	−0·3 Δk	4
Brisbane 6229	+0·020	+0·2 Δk	4	Cape 1880, 2901	−0·004	−0·3 Δk	3
Lacaille 7442	+0·017	0·0 Δk	5	Lacaille 2512	−0·029	−0·2 Δk	3
Lacaille 7751	0·000	−0·1 Δk	3	Lacaille 3274	−0·005	+0·1 Δk	2
σ Octantis	+0·012	−0·2 Δk	3	Gilliss P.Z. 5752	−0·046	+0·4 Δk	1
Gilliss P.Z. 13504	+0·046	−0·4 Δk	1	A Octantis	−0·031	+0·3 Δk	3
Lacaille 8094	+0·002	−0·3 Δk	3	Brisbane 2007	−0·040	+0·4 Δk	3
Lacaille 8257	+0·006	−0·5 Δk	2	θ Chamæleontis	−0·055	+0·5 Δk	2
C.G.A. 28663	+0·024	−0·7 Δk	2	Lacaille 3759	−0·013	+0·6 Δk	2
C.G.A. 28714	−0·006	−0·9 Δk	2	ζ Octantis	−0·040	+0·8 Δk	1
Lacaille 8551	+0·004	−0·9 Δk	1	Lacaille 4342	−0·014	+1·0 Δk	1
Lacaille 8751	−0·001	−0·9 Δk	1	C.G.A. 14481	−0·016	+1·1 Δk	1
Lacaille 8720	−0·037	−0·9 Δk	1	Lacaille 4510	−0·005	+1·1 Δk	1
B Octantis	+0·015	−0·9 Δk	1	Lacaille 4578	−0·027	+1·0 Δk	1
Lacaille 8738	+0·007	−0·9 Δk	1	η Octantis	−0·019	+1·1 Δk	1
υ (C.) Octantis	−0·018	−1·1 Δk	1	Lacaille 4708	−0·075	+1·1 Δk	1
τ Octantis	+0·008	−1·1 Δk	1	Lacaille 4731	−0·028	+1·1 Δk	1
Lacaille 9464	+0·008	−1·0 Δk	1	C.G.A. 15761	−0·042	+1·0 Δk	1
Lacaille 9563	−0·002	−1·1 Δk	1	Lacaille 4865	−0·038	+1·0 Δk	1
Lacaille 9596	−0·006	−1·1 Δk	1	Lacaille 4991	−0·014	+1·0 Δk	1
	+0·010	−0·38 Δk,	39		−0·030	+0·40 Δk,	34

The means, taken with weight proportional to the number of observations of each star, give a revised determination of the polar correction, Δp, as originally applied during the interval Sept. 4–11. From a combination of the above- and below-pole results we derive

$$\Delta p = +0^{s}\cdot019 - 0\cdot39\Delta k$$

as compared with the value $+0^{s}\cdot018 - 0\cdot46\Delta k$ contained in Table II.

Subtracting this value from those derived from the separate stars, the following equations were derived which were regarded as equations of condition for the determination of Δk. In this application they are regarded as expressing the condition that, apart from accidental variation, the true value of Δp is sensibly constant throughout the period Sept. 4–11.

TABLE VI.

Equations of Condition for Period September 4–11.

	s			s	
χ Octantis	+0·004	+0·6 $\Delta k=0$	Lacaille 2296	−0·030	−0·7 $\Delta k=0$
Brisbane 6229	+0·001	+0·6 $\Delta k=0$	Cape 1880, 2901	+0·015	−0·7 $\Delta k=0$
Lacaille 7442	−0·002	+0·4 $\Delta k=0$	Lacaille 2512	−0·010	−0·6 $\Delta k=0$
Lacaille 7751	−0·019	+0·3 $\Delta k=0$	Lacaille 3274	+0·014	−0·3 $\Delta k=0$
σ Octantis	−0·007	+0·2 $\Delta k=0$	Gilliss P.Z. 5752	−0·027	0·0 $\Delta k=0$
Gilliss P.Z. 13504	+0·027	0·0 $\Delta k=0$	A Octantis	−0·012	−0·1 $\Delta k=0$
Lacaille 8094	−0·017	+0·1 $\Delta k=0$	Brisbane 2007	−0·021	0·0 $\Delta k=0$
Lacaille 8257	−0·013	−0·1 $\Delta k=0$	θ Chamæleontis	−0·036	+0·1 $\Delta k=0$
C.G.A. 28663	+0·005	−0·3 $\Delta k=0$	Lacaille 3759	+0·006	+0·2 $\Delta k=0$
C.G.A. 28714	−0·025	−0·5 $\Delta k=0$	ζ Octantis	−0·021	+0·4 $\Delta k=0$
Lacaille 8551	−0·015	−0·5 $\Delta k=0$	Lacaille 4342	+0·005	+0·6 $\Delta k=0$
Lacaille 8751	−0·020	−0·5 $\Delta k=0$	C.G.A. 14481	+0·003	+0·7 $\Delta k=0$
Lacaille 8720	−0·056	−0·5 $\Delta k=0$	Lacaille 4510	+0·014	+0·7 $\Delta k=0$
B Octantis	−0·004	−0·5 $\Delta k=0$	Lacaille 4578	−0·008	+0·6 $\Delta k=0$
Lacaille 8738	−0·012	−0·5 $\Delta k=0$	η Octantis	0·000	+0·7 $\Delta k=0$
υ (C.) Octantis	−0·037	−0·7 $\Delta k=0$	Lacaille 4708	−0·056	+0·7 $\Delta k=0$
τ Octantis	−0·011	−0·7 $\Delta k=0$	Lacaille 4731	−0·009	+0·7 $\Delta k=0$
Lacaille 9464	−0·011	−0·6 $\Delta k=0$	C.G.A. 15761	−0·023	+0·6 $\Delta k=0$
Lacaille 9563	−0·021	−0·7 $\Delta k=0$	Lacaille 4865	−0·019	+0·6 $\Delta k=0$
Lacaille 9596	−0·025	−0·7 $\Delta k=0$	Lacaille 4991	+0·005	+0·6 $\Delta k=0$

Forming normal equations from these equations of condition, with weights proportional to the number of observations of each star, we find

from above-pole observations	$8\cdot44\Delta k + 0^{s}\cdot145 = 0$
from below-pole observations	$9\cdot23\Delta k - 0^{s}\cdot016 = 0$
from all observations	$17\cdot67\Delta k + 0^{s}\cdot129 = 0.$

Similar normals have been derived from each group. In the following table the results from the separate groups are given in the form of normal equations which may be combined by addition.

TABLE VII.

Normal Equations from Separate Groups of Right Ascension.

	s	
Jan. 16–26	−0·038	+ 1·5 $\Delta k=0$
Jan. 28–Feb. 8	−0·003	+ 1·3 $\Delta k=0$
Feb. 16–23	+0·029	+ 0·5 $\Delta k=0$
Feb. 24–Mar. 3	+0·031	+ 0·3 $\Delta k=0$
Mar. 13–20	−0·019	+ 0·3 $\Delta k=0$
Mar. 21–30	+0·041	+ 0·6 $\Delta k=0$
Apr. 1–8	+0·176	+ 7·3 $\Delta k=0$
Apr. 12–24	+0·118	+ 5·7 $\Delta k=0$
Apr. 28–May 12	−0·204	+12·5 $\Delta k=0$
May 14–27	+0·056	+ 7·0 $\Delta k=0$
May 30–June 9	+0·093	+ 5·5 $\Delta k=0$
June 10–21	+0·149	+12·7 $\Delta k=0$
June 25–July 3	−0·156	+11·0 $\Delta k=0$
July 4–19	−0·221	+11·1 $\Delta k=0$
July 20–Aug. 8	−0·056	+ 6·7 $\Delta k=0$
Aug. 12–26	+0·019	+13·4 $\Delta k=0$
Aug. 28–Sept. 2	−0·012	+ 9·0 $\Delta k=0$
Sept. 4–11	+0·129	+17·7 $\Delta k=0$
Sept. 12–19	+0·040	+16·7 $\Delta k=0$
Sept. 28–Oct. 7	+0·317	+25·1 $\Delta k=0$
Oct. 9–20	−0·318	+17·0 $\Delta k=0$
Oct. 25–28	+0·216	+13·9 $\Delta k=0$
Oct. 30–Nov. 6	−0·140	+12·2 $\Delta k=0$
Nov. 7–12	+0·090	+ 4·5 $\Delta k=0$
Nov. 13–19	−0·035	+ 5·5 $\Delta k=0$
Nov. 20–29	−0·121	+ 8·9 $\Delta k=0$
	+0ˢ·181	+227·9 $\Delta k=0$

From the normals as thus combined we therefore derive

$$\Delta k = -0^{s}\cdot0008 \pm 0^{s}\cdot0020$$
$$= -0''\cdot012 \pm 0''\cdot030.$$

The probable error here quoted is derived from the final factor of Δk, as indicating its weight referred to a unit weight derived from the discordances between the indications of the separate epochs.

The analysis also yields the following revised determinations of the instrumental polar corrections for the different epochs.

TABLE VIII.

Instrumental Polar Corrections in R.A.

Epoch.	Δp.		Inst.
	s		
Jan. 16–26	−0·030	−1·28 Δk	W.
Jan. 28–Feb. 8	−0·015	−1·14 Δk	E.
Feb. 16–23	−0·026	−1·05 Δk	W.
Feb. 24–Mar. 3	+0·008	−0·95 Δk	E.
Mar. 13–20	+0·014	−0·79 Δk	E.
Mar. 21–30	−0·057	−0·69 Δk	W.
Apr. 1–8	+0·028	−0·52 Δk	E.
Apr. 12–24	−0·029	−0·37 Δk	W.
Apr. 28–May 12	+0·013	−0·27 Δk	E.
May 14–27	−0·015	−0·08 Δk	W.
May 30–June 9	+0·036	−0·12 Δk	E.
June 10–21	+0·005	−0·12 Δk	W.
June 25–July 3	+0·030	−0·09 Δk	E.
July 4–19	+0·017	−0·04 Δk	W.
July 20–Aug. 8	+0·043	−0·04 Δk	E.
Aug. 12–26	+0·030	0·00 Δk	W.
Aug. 28–Sept. 2	+0·013	−0·03 Δk	E.
Sept. 4–11	+0·019	−0·39 Δk	W.
Sept. 12–19	+0·019	−0·53 Δk	E.
Sept. 28–Oct. 7	+0·013	−0·74 Δk	W.
Oct. 9–20	+0·029	−0·83 Δk	E.
Oct. 25–28	+0·015	−0·86 Δk	W.
Oct. 30–Nov. 6	+0·033	−0·87 Δk	E.
Nov. 7–12	0·000	−0·91 Δk	W.
Nov. 13–19	+0·032	−1·19 Δk	E.
Nov. 20–29	+0·007	−1·18 Δk	W.

Assuming that these are primarily due to changes in the azimuth of the meridian marks produced by the motion of the earth's axis of rotation within the earth itself, we may equate them to the values of the latter derived from the investigations of the international latitude stations. Combining pairs of adjacent determinations taken in opposite clamps, in order more efficiently to eliminate any outstanding errors in the adopted collimation correction, and introducing a constant term ΔA which may arise from an erroneous value in the adopted mean azimuth of the marks, we thus derive the following twenty-four equations of condition.

TABLE IX.

Equations of Condition from Comparison with International Latitude Results (R.A.).

Date.		O.	C.
		s	s
1911·07	$\Delta A - 1{\cdot}21\,\Delta k$	$-0{\cdot}023$	$= -0{\cdot}015$
·11	$\Delta A - 1{\cdot}10\,\Delta k$	$-0{\cdot}021$	$= -0{\cdot}016$
·15	$\Delta A - 1{\cdot}00\,\Delta k$	$-0{\cdot}009$	$= -0{\cdot}013$
·21	$\Delta A - 0{\cdot}74\,\Delta k$	$-0{\cdot}022$	$= -0{\cdot}010$
·24	$\Delta A - 0{\cdot}61\,\Delta k$	$-0{\cdot}015$	$= -0{\cdot}006$
·28	$\Delta A - 0{\cdot}45\,\Delta k$	$-0{\cdot}001$	$= -0{\cdot}002$
·32	$\Delta A - 0{\cdot}32\,\Delta k$	$-0{\cdot}008$	$= +0{\cdot}002$
·36	$\Delta A - 0{\cdot}18\,\Delta k$	$-0{\cdot}001$	$= +0{\cdot}007$
·40	$\Delta A - 0{\cdot}10\,\Delta k$	$+0{\cdot}011$	$= +0{\cdot}012$
·44	$\Delta A - 0{\cdot}12\,\Delta k$	$+0{\cdot}021$	$= +0{\cdot}015$
·47	$\Delta A - 0{\cdot}10\,\Delta k$	$+0{\cdot}018$	$= +0{\cdot}018$
·50	$\Delta A - 0{\cdot}07\,\Delta k$	$+0{\cdot}024$	$= +0{\cdot}021$
·55	$\Delta A - 0{\cdot}04\,\Delta k$	$+0{\cdot}030$	$= +0{\cdot}023$
·61	$\Delta A - 0{\cdot}02\,\Delta k$	$+0{\cdot}037$	$= +0{\cdot}025$
·65	$\Delta A - 0{\cdot}02\,\Delta k$	$+0{\cdot}022$	$= +0{\cdot}024$
·67	$\Delta A - 0{\cdot}21\,\Delta k$	$+0{\cdot}016$	$= +0{\cdot}024$
·69	$\Delta A \quad 0{\cdot}46\,\Delta k$	$+0{\cdot}019$	$= +0{\cdot}024$
·72	$\Delta A - 0{\cdot}64\,\Delta k$	$+0{\cdot}016$	$= +0{\cdot}023$
·77	$\Delta A - 0{\cdot}79\,\Delta k$	$+0{\cdot}021$	$= +0{\cdot}020$
·81	$\Delta A - 0{\cdot}85\,\Delta k$	$+0{\cdot}022$	$= +0{\cdot}017$
·83	$\Delta A - 0{\cdot}87\,\Delta k$	$+0{\cdot}024$	$= +0{\cdot}015$
·85	$\Delta A - 0{\cdot}89\,\Delta k$	$+0{\cdot}017$	$= +0{\cdot}014$
·86	$\Delta A - 1{\cdot}05\,\Delta k$	$+0{\cdot}016$	$= +0{\cdot}013$
1911·88	$\Delta A - 1{\cdot}19\,\Delta k$	$+0{\cdot}020$	$= +0{\cdot}011$

The absolute terms (O) on the left are those derived from the present investigation. Those on the right (C) depend on Albrecht's determination of the polar motion. The latter are derived from smoothed curves, and it is difficult either to assign any suitable variation of weight among themselves or to compare their absolute accuracy with that of the quantities O. Hence it seemed best to combine the observations with equal weight. This yielded the following normals:

$$24\Delta A - 13{\cdot}03\Delta k = -0^{s}{\cdot}012$$
$$-13{\cdot}03\Delta A + 10{\cdot}97\Delta k = -0^{s}{\cdot}009$$

whence $\Delta k = -0^{s}{\cdot}0008$, weight 3·9.

The probable error corresponding to unit weight, as dependent on the residuals in the least square solution, is

$$\pm 0^s\cdot 0047.$$

Thus finally

$$\Delta k = -0^s\cdot 0008 \pm 0^s\cdot 0024$$
$$= -0''\cdot 012 \pm 0''\cdot 036.$$

The declination observations have been treated in an exactly analogous manner, the latitude corrections playing a role equivalent to that of the instrumental polar corrections in right ascension. The following are the normal equations resulting from the separate groups.

TABLE X.

Normal Equations from Separate Groups of Declinations.

Jan. 16–26	$27\cdot 4\,\Delta k$	$+ 2''\cdot 02 = 0$
Jan. 28–Feb. 8	$43\cdot 4\,\Delta k$	$+ 9\cdot 15 = 0$
Feb. 16–23	$48\cdot 5\,\Delta k$	$+ 12\cdot 90 = 0$
Feb. 24–Mar. 3	$28\cdot 4\,\Delta k$	$+ 6\cdot 14 = 0$
Mar. 13–20	$11\cdot 2\,\Delta k$	$+ 2\cdot 86 = 0$
Mar. 21–30	$15\cdot 8\,\Delta k$	$- 1\cdot 44 = 0$
Apr. 1–8	$11\cdot 9\,\Delta k$	$+ 8\cdot 48 = 0$
Apr. 12–24	$9\cdot 4\,\Delta k$	$- 0\cdot 78 = 0$
Apr. 28–May 12	$12\cdot 6\,\Delta k$	$+ 3\cdot 86 = 0$
May 14–27	$3\cdot 0\,\Delta k$	$+ 0\cdot 45 = 0$
May 30–June 9	$1\cdot 3\,\Delta k$	$+ 1\cdot 42 = 0$
June 10–21	$4\cdot 5\,\Delta k$	$- 1\cdot 50 = 0$
June 25–July 3	$2\cdot 6\,\Delta k$	$+ 1\cdot 02 = 0$
July 4–19	$1\cdot 9\,\Delta k$	$- 0\cdot 88 = 0$
July 20–Aug. 8	$1\cdot 4\,\Delta k$	$- 0\cdot 73 = 0$
Aug. 12–26	$11\cdot 2\,\Delta k$	$- 3\cdot 07 = 0$
Aug. 28–Sept. 2	$10\cdot 9\,\Delta k$	$+ 0\cdot 65 = 0$
Sept. 4–11	$6\cdot 8\,\Delta k$	$- 0\cdot 20 = 0$
Sept. 12–19	$5\cdot 4\,\Delta k$	$- 2\cdot 39 = 0$
Sept. 28–Oct. 7	$11\cdot 3\,\Delta k$	$- 1\cdot 67 = 0$
Oct. 9–20	$9\cdot 4\,\Delta k$	$+ 1\cdot 90 = 0$
Oct. 25–28	$12\cdot 3\,\Delta k$	$+ 9\cdot 28 = 0$
Oct. 30–Nov. 6	$9\cdot 1\,\Delta k$	$- 0\cdot 86 = 0$
Nov. 7–12	$4\cdot 4\,\Delta k$	$+ 2\cdot 00 = 0$
Nov. 13–19	$33\cdot 6\,\Delta k$	$+ 4\cdot 71 = 0$
Nov. 20–29	$34\cdot 9\,\Delta k$	$- 3\cdot 13 = 0$
	$372\cdot 6\,\Delta k$	$+ 50\cdot 19 = 0$

with the solution

$$\Delta k = -0''\cdot 135 \pm 0''\cdot 033.$$

The revised corrections to the adopted instrumental latitude for the different epochs are as follows :—

TABLE XI.

Corrections to Instrumental Latitude.

Epoch.	$\Delta\phi$.		Inst.
Jan. 16–26	+0".27	−1·06 Δk	W.
Jan. 28–Feb. 8	−0·40	−0·96 Δk	E.
Feb. 16–23	−0·01	−0·37 Δk	W.
Feb. 24–Mar. 3	−0·27	−0·45 Δk	E.
Mar. 13–20	−0·40	+0·05 Δk	E.
Mar. 21–30	−0·50	+0·14 Δk	W.
Apr. 1–8	−0·80	0·00 Δk	E.
Apr. 12–24	−0·57	+0·01 Δk	W.
Apr. 28–May 12	−0·48	+0·05 Δk	E.
May 14–27	−0·11	−0·01 Δk	W.
May 30–June 9	−0·45	−0·02 Δk	E.
June 10–21	−0·53	+0·04 Δk	W.
June 25–July 3	−0·31	+0·03 Δk	E.
July 4–19	−0·12	+0·02 Δk	W.
July 20–Aug. 8	0·00	−0·03 Δk	E.
Aug. 12–26	−0·03	−0·34 Δk	W.
Aug. 28–Sept. 2	−0·17	−0·62 Δk	E.
Sept. 4–11	+0·06	−0·92 Δk	W.
Sept. 12–19	−0·09	−1·10 Δk	E.
Sept. 28–Oct. 7	+0·14	−1·25 Δk	W.
Oct. 9–20	+0·05	−1·35 Δk	E.
Oct. 25–28	+0·05	−1·38 Δk	W.
Oct. 30–Nov. 6	−0·03	−1·43 Δk	E.
Nov. 7–12	+0·27	−1·36 Δk	W.
Nov. 13–19	−0·05	−0·96 Δk	E
Nov. 20–29	+0·26	−1·16 Δk	W.

A constant difference between results obtained with clamp E and clamp W may result from the fact that different parts of the divided circle are brought into use in the neighbourhood of the pole in the two positions. This difference may be eliminated by combining into means the results of adjacent groups in opposite clamps. Retaining also a constant $\Delta\phi_0$ to represent a constant error in the adopted mean latitude, we form the equations of condition.

TABLE XII.

Equations of Condition from Comparison with International Latitude Results (Dec.).

Date.		O.	C.
1911·07	$\Delta\phi_0 - 1{\cdot}01\,\Delta k$	$-0{\cdot}06$	$= -0''{\cdot}12$
·11	$\Delta\phi_0 - 0{\cdot}67\,\Delta k$	$-0{\cdot}21$	$= -0{\cdot}15$
·15	$\Delta\phi_0 - 0{\cdot}41\,\Delta k$	$-0{\cdot}14$	$= -0{\cdot}19$
·21	$\Delta\phi_0 + 0{\cdot}10\,\Delta k$	$-0{\cdot}45$	$= -0{\cdot}26$
·24	$\Delta\phi_0 + 0{\cdot}07\,\Delta k$	$-0{\cdot}65$	$= -0{\cdot}28$
·28	$\Delta\phi_0 + 0{\cdot}01\,\Delta k$	$-0{\cdot}68$	$= -0{\cdot}32$
·32	$\Delta\phi_0 + 0{\cdot}03\,\Delta k$	$-0{\cdot}53$	$= -0{\cdot}35$
·36	$\Delta\phi_0 + 0{\cdot}02\,\Delta k$	$-0{\cdot}30$	$= -0{\cdot}36$
·40	$\Delta\phi_0 - 0{\cdot}02\,\Delta k$	$-0{\cdot}28$	$= -0{\cdot}38$
·44	$\Delta\phi_0 + 0{\cdot}01\,\Delta k$	$-0{\cdot}49$	$= -0{\cdot}38$
·47	$\Delta\phi_0 + 0{\cdot}04\,\Delta k$	$-0{\cdot}42$	$= -0{\cdot}38$
·50	$\Delta\phi_0 + 0{\cdot}03\,\Delta k$	$-0{\cdot}22$	$= -0{\cdot}37$
·55	$\Delta\phi_0 - 0{\cdot}01\,\Delta k$	$-0{\cdot}06$	$= -0{\cdot}31$
·61	$\Delta\phi_0 - 0{\cdot}19\,\Delta k$	$-0{\cdot}02$	$= -0{\cdot}23$
·65	$\Delta\phi_0 - 0{\cdot}48\,\Delta k$	$-0{\cdot}10$	$= -0{\cdot}18$
·67	$\Delta\phi_0 - 0{\cdot}77\,\Delta k$	$-0{\cdot}05$	$= -0{\cdot}16$
·69	$\Delta\phi_0 - 1{\cdot}01\,\Delta k$	$-0{\cdot}02$	$= -0{\cdot}13$
·72	$\Delta\phi_0 - 1{\cdot}18\,\Delta k$	$+0{\cdot}03$	$= -0{\cdot}09$
·77	$\Delta\phi_0 - 1{\cdot}30\,\Delta k$	$+0{\cdot}10$	$= -0{\cdot}03$
·81	$\Delta\phi_0 - 1{\cdot}37\,\Delta k$	$+0{\cdot}05$	$= +0{\cdot}02$
·83	$\Delta\phi_0 - 1{\cdot}41\,\Delta k$	$+0{\cdot}01$	$= +0{\cdot}04$
·85	$\Delta\phi_0 - 1{\cdot}40\,\Delta k$	$+0{\cdot}12$	$= +0{\cdot}06$
·86	$\Delta\phi_0 - 1{\cdot}16\,\Delta k$	$+0{\cdot}11$	$= +0{\cdot}07$
1911·88	$\Delta\phi_0 - 1{\cdot}06\,\Delta k$	$+0{\cdot}11$	$= +0{\cdot}09$

Here the quantities O are derived from the series of observations under discussion, while the quantities C depend on the investigations of the international latitude stations. Combining the equations with equal weights we obtain the normals

$$24{\cdot}00\Delta\phi_0 - 13{\cdot}14\Delta k = -0''{\cdot}23$$
$$-13{\cdot}14\Delta\phi_0 + 14{\cdot}89\Delta k = +0''{\cdot}45$$

whence $\Delta k = +0''{\cdot}041$, weight 7·74.

The probable error corresponding to unit weight derived from the residuals is $\pm 0''{\cdot}11$, and that of the derived value of Δk is therefore $\pm 0''{\cdot}034$.

We thus have the following four virtually independent determinations of the aberration constant:—

(1) from observations of right ascension,

$$k = 20''{\cdot}46 \pm 0''{\cdot}030\,;$$

(2) from observations of azimuth, compared with Albrecht's determination of the polar motion,

$$k = 20''{\cdot}46 \pm 0''{\cdot}036\,;$$

(3) from declination observations,

$$k = 20''{\cdot}33 \pm 0''{\cdot}033\,;$$

(4) from latitude determinations in combination with Albrecht's determination of the polar motion,

$$k = 20''{\cdot}51 \pm 0''{\cdot}034.$$

The simple mean of these four results gives

$$k = 20''{\cdot}44 \pm 0''{\cdot}017,$$

where the probable error is estimated from the internal agreement of the constituent material of the four separate results. The discordances between the results themselves point to a somewhat higher value of this probable error, chiefly on account of the outstanding determination (3). The actual result will not be materially changed by any reasonable alteration of the combining weights. Supposing, however, that each result is liable to its own peculiar systematic errors, we may derive the probable error of the combination from the agreement of the four separate determinations as

$$\pm 0''{\cdot}026.$$

The final value of k ($20''{\cdot}44$) is somewhat smaller than that derived from the preliminary discussion of the same material contained in the *Cape Annals*.

The erroneous computation of the coefficients of Δk, referred to above, will tend to modify the determinations (1) and (4) of that discussion in favour of the two determinations (2) and (4) of the present one. It appears also that in the determinations (2) and (3) of the former paper, the quantities which figure as corrections to the tabular aberration constant were interpreted with erroneous sign. Hence these determinations should be replaced by

$$k = 20''{\cdot}36,\ k = 20''{\cdot}465.$$

These two determinations, with additional material included, are now replaced by the determination (3) of the present paper.

Suggested re-arrangement of Observations for the Determination of the Aberration Constant.

The aberration factors in right ascension involve the expression $\sin(H + \alpha)$ associated with a negative sign.

Hence at any epoch they will be greatest for those stars for which $H + \alpha = 18$ hours, and least for those stars for which $H + \alpha = 6$ hours. But we have approximately, on neglecting the obliquity of the ecliptic and the eccentricity of the Earth's orbit,

$$H = 18 \text{ hours.} - \odot,$$

where $\odot$ denotes the Sun's longitude. Hence the maximum aberration factors will occur with stars for which

$$\alpha = \odot,$$

and the minimum with stars for which

$$\alpha = \odot - 12 \text{ hours.}$$

It follows that the most favourable times for observing transits for the determination of the aberration constant are near midday and midnight.

A star for which α approximates to $\odot$ will reach its upper culmination near midday and its lower culmination near midnight; and if both culminations could be observed, the effects of an error in the aberration constant would be entirely eliminated from the resulting determination of azimuth, but thrown with full effect into the determination of R.A. We should have, in fact, from the mean of the two observations,

$$\text{true R.A.} = \text{observed R.A.} + \Delta k \sec \delta \quad . \quad . \quad (a)$$

After an interval of six months, when the value of $\odot$ has increased by 12 hours, the same star will again present itself for observation at midday and midnight, but in reversed culminations, *i.e.* the upper culmination would occur at midnight and the lower culmination at midday. In this case, both at upper and lower culminations, the aberration correction will again have its maximum value, but with the reversed sign. For the second epoch we have

$$\text{true R.A.} = \text{observed R.A.} - \Delta k \sec \delta \quad . \quad . \quad (b)$$

Denoting the observed R.A.'s at the two epochs by α_1, α_2, the combinations of the two equations (a) (b) gives

$$2\Delta k = (\alpha_2 - \alpha_1) \cos \delta.$$

If the same process be repeated with a number of stars, the weight of the final determination would depend on the number of adjacent days and nights on which each star can be observed sufficiently near to midnight and midday, say within one hour or one hour and a half, and also on the number of stars used. By

making observations at three consecutive epochs of maximum aberration effect instead of two, we could provide for the elimination of any errors of proper motion, while, as changes due to parallax have the same period as those due to aberration, but with complementary phase, the parallax factors throughout will approximate to zero.

The process is, however, inadmissible, as it would require that at least half the observations should be made by daylight, and in the southern hemisphere there are no stars in the immediate vicinity of the pole sufficiently bright to permit of daylight observing. In this latitude consecutive upper and lower transits can be observed during about four months in the year, but then only near the times of sunrise and sunset, when the aberration factor approximates to zero. Recourse must therefore be had to less direct methods. In the light of the analysis of the previous observations, and having regard to the conditions indicated which yield the largest aberration factors, the following process is suggested.

Consider the stars involved divided into four groups centred about R.A.'s 0^h, 6^h, 12^h, 18^h, but which in part overlap.

Group I. consists of stars which at one epoch can be observed at lower transit in the evening and at upper transit in the morning. These stars are included roughly between R.A. $20\frac{1}{2}$ hours and R.A. $3\frac{1}{2}$ hours.

Group II. consists of stars having right ascensions between the approximate limits $1\frac{1}{2}$ hours and $10\frac{1}{2}$ hours, which can be observed at lower culmination at or near midnight at a time of year when azimuths can be derived from consecutive transits, *i.e.* between the middle of April and the middle of August.

Group III. contains stars which can be observed at the upper transit in the evening and at lower transit in the morning at one and the same epoch. The limiting right ascensions are about $8\frac{1}{2}$ hours and $15\frac{1}{2}$ hours.

Group IV. contains stars whose upper culmination occurs near midnight within the period during which the consecutive transits can be observed. The limiting R.A.'s are about $13\frac{1}{2}$ hours and $22\frac{1}{2}$ hours.

If now the observations of any one night during the winter months be divided into three short watches (distributed round six hours before midnight, midnight, and six hours after midnight, when the same series of stars can be observed in the first and third of these watches), from the *mean* of the evening and morning observations of R.A. we may derive the true R.A. of stars of Groups I. and III. independent of azimuth error, provided only that the azimuth of the meridian marks remains sensibly constant through the night. On the other hand, the *difference* between the upper and lower culminations for each star observed at both culminations will yield a determination of azimuth, or rather of instrumental polar error, in the form

$$\Delta p = \tfrac{1}{2}(\alpha_{\text{above}} - \alpha_{\text{below}}) \sec \delta.$$

This determination is quite independent of the tabular right ascensions, and also of any aberration effects except those due to diurnal aberration, which may be allowed for in the usual way.

During the midnight watch stars which reach their lower culmination will belong to Group II., and those which reach their upper culmination belong to Group IV. In reducing these observations the instrumental correction Δp must be supplied from the evening and morning observations, and the further reductions from apparent to mean place require corrections $+\Delta k \sec \delta$ for stars of Group II. and $-\Delta k \sec \delta$ for stars of Group IV.

In other words, the mean right ascensions from the midnight observations, derived with the tabular value of the aberration constant, may be regarded as determinations of

$$\alpha_{II} - \Delta k \sec \delta, \quad \alpha_{IV} + \Delta k \sec \delta,$$

where α_{II} or α_{IV} denotes a typical *mean* R.A. of a star belonging to Group II. or Group IV.

Thus, from observations during the winter months alone, we may derive values of

$$\alpha_I, \quad \alpha_{II} - \Delta k \sec \delta, \quad \alpha_{III}, \quad \alpha_{IV} + \Delta k \sec \delta$$

for each star involved in Groups I., II., III., IV. respectively.

In so far as consecutive groups overlap, the differences between the determinations for the stars common to two groups will in all cases yield a determination of Δk with a factor $\pm$ 1. This factor is relatively small, and the restricted ranges of R.A. common to consecutive groups will limit the number of suitable stars available for such direct use. It is necessary, therefore, to consider how best the determinations may be strengthened by observations taken at epochs of the year when the stars available for observation can be seen at a single culmination only.

At these times the star places, as derived by the preceding processes dependent on the winter observations, may be regarded as tabular places to be used in conjunction with the additional observations for the determination of the instrumental polar correction at the instant of each of these additional observations. This polar correction is computed by the formula

$$\Delta p = (\alpha_o - \alpha_c) \cos \delta \quad . \quad . \quad . \quad . \quad \text{(A)}$$

for stars at upper culmination, or

$$\Delta p = (\alpha_c - \alpha_o) \cos \delta \quad . \quad . \quad . \quad . \quad \text{(B)}$$

for stars at lower culmination; where α_o denotes an observed R.A. and α_c the corresponding tabular R.A., both being considered as computed with the tabular value of the aberration constant.

Now both α_o, α_c as here involved will be liable to error in consequence of an erroneous value of the aberration constant used in reduction. (The comparison is supposed to be made through the reduced *mean* places.) This error will vary from star to star, and consequently an erroneous aberration constant will exhibit

itself as an apparent change in the polar error in the course of a night.

If the tabular places α_c were accurately known, the correction to Δp as derived by the formulæ (A) or (B) (*i.e.* either from upper or lower culminations) will go through approximately the same sequence of changes each day, ranging from $-\Delta k$ at midday to $+\Delta k$ at midnight.

By comparing determinations at the extreme range available we shall secure the most favourable factors for determination of Δk—*i.e.* it is desirable that observations should be made at midnight on the one hand, and as near to dusk or dawn on the other as the intervention of daylight will permit.

The requisite conditions would be attained if all the stars concerned, whether observable at midnight, or at dusk or dawn, were included in Groups I. and III., since we have seen that the tabular mean places for such stars can be derived freed from errors in aberration.

At midsummer, however, the stars which present themselves for observation at midnight belong to Groups II. and IV., Group II. being available for above-pole determinations and Group IV. for below-pole determinations. For these stars the tabular right ascensions, as derived from observations associated with double transits in the winter months, require corrections $+\Delta k \sec\delta$ for upper culminations and $-\Delta k \sec\delta$ for lower culminations. Hence the value of Δp derived from midnight observations will require an additional correction Δk, making a total correction $+2\Delta k$.

The dusk and dawn observations will depend on stars belonging to Groups I. and III. The error in Δp resulting from an erroneous aberration constant will therefore be solely that introduced through the medium of the observed quantity α_o. The factor associated with Δk for these observations will be small, but (considered as a correction to Δp) of the same sign as that involved in midnight observations. In the most unfavourable circumstances (at exact midsummer when the nights are shortest) the least value of this factor for this latitude will be about 0·5. We thus have

true polar correction = polar correction from midnight observations $+ 2\Delta k$

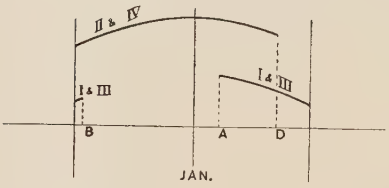
true polar correction = polar correction from dusk or dawn observations $+ 0{\cdot}5\Delta k$,

whence, if the true polar correction may be considered invariable

$1{\cdot}5\Delta k$ = (polar correction from dusk or dawn observations) − (polar correction from midnight observations).

As the night lengthens, on either side of midsummer, the range of factor will increase and more favourable determinations of Δk will be possible so long as the stars presenting themselves at dusk, midnight, and dawn belong to the same groups as above.

Still more favourable will be the circumstances when stars



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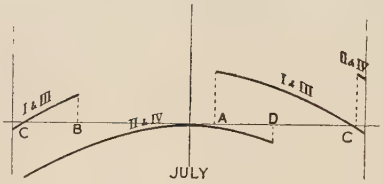

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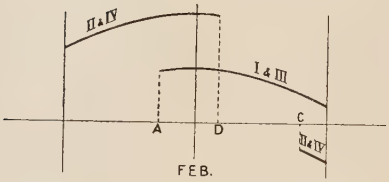

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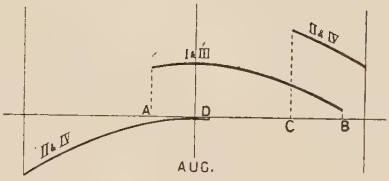

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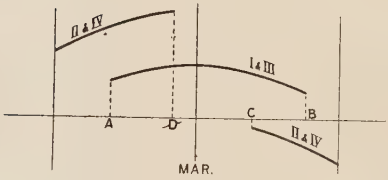

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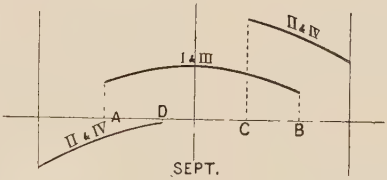

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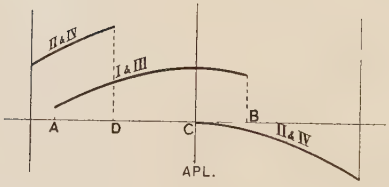

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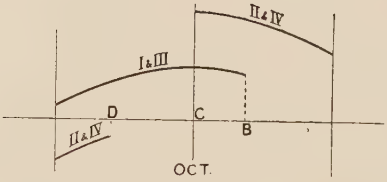

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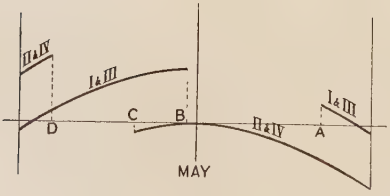

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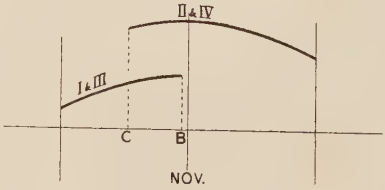

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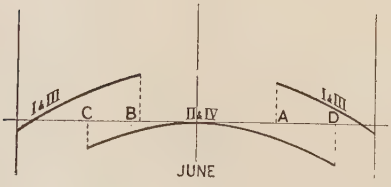

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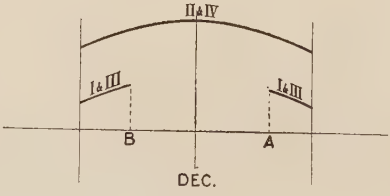


DEC.

Aberration Factors in Azimuth.

belonging to Groups II. and IV. are available for either evening or morning determinations with the same groups, but at opposite culminations, at or near midnight. These circumstances will occur at intermediate times between midsummer and midwinter. For the dusk or dawn observations of stars of Groups II. and IV. the factor associated with Δk, in Δp, must be diminished by unity on account of the error introduced through the medium of the adopted tabular place. The factor introduced through the medium of the observation will be small and generally positive, say about 0·2. Hence the total factor for the dusk or dawn observation will be about $-0{\cdot}8$ as compared with $+2{\cdot}0$ for the midnight observation, giving a total range in Δp for a night's observing of $2{\cdot}8\Delta k$.

The most favourable circumstances for each month in the year are indicated graphically in the annexed diagram, Plate 4, the different figures in which illustrate the circumstances as computed for the latitude of the Cape at about the middle of each month.

The abscissæ represent times of the night, the central ordinate corresponding in all cases with midnight, and the extreme ordinates with the times when the Sun is 6° below the horizon.

The curves, which all form parts of curves of sines, represent the aberration factors in Δp in the sense in which they must be applied as corrections to the observed values, derived through the medium of tabular places dependent on the winter observations, and reduced with the tabular value of the aberration constant. The discontinuities in the curve result from the method in which the tabular places are derived, whether from dusk or dawn observations (Groups I. and III.) or from midnight observations (Groups II. and IV.).

Groups I. and III. are regarded as commencing with the star Lacaille 8257, which transits above pole at $20^h\ 23^m$ and below pole at $8^h\ 23^m$, and ending with Lacaille 1414, which transits above pole at $3^h\ 40^m$ and below pole at $15^h\ 40^m$. These times are indicated on the figures by the ordinates marked A and B.

Groups II. and IV. start with Gilliss P.Z. 1219 at $1^h\ 30^m$ or $13^h\ 30^m$, and finish with Lacaille 9102 at $10^h\ 30^m$ or $22^h\ 30^m$. The times of transit of these stars are indicated by the ordinates marked C and D.

The most favourable times of observing at any epoch are those which approximate to the greatest and least ordinates in the corresponding figure. Where one of these extreme ordinates intersects two branches, the particular branch which yields the greatest or least factor is clearly seen, and the tabular place to be used is that of the star derived as a member of the corresponding group.

Similar figures are also given, Plate 5, for the declination observations, in which latitude determinations play an analogous role to the quantity Δp involved in the right ascensions. Here,

however, the factor introduced through the medium of the observation itself decreases continuously from dusk to dawn, without passing through a maximum or minimum value, while at midsummer the additional factors introduced through the medium of the tabular position as derived in winter combine with this in a cumulative direction. The tabular declinations of Stars of Groups II. and IV. will be free from aberration error, while those of stars in Groups I. and III. will be in error by an amount $\pm\Delta k$. The total variation in apparent latitude between dusk and dawn at midsummer amounts to about $3{\cdot}6\Delta k$. This factor will, however, rapidly decrease as the winter is approached. On the whole, an increase in latitude would be more favourable for R.A. determinations and less favourable for declination determinations.

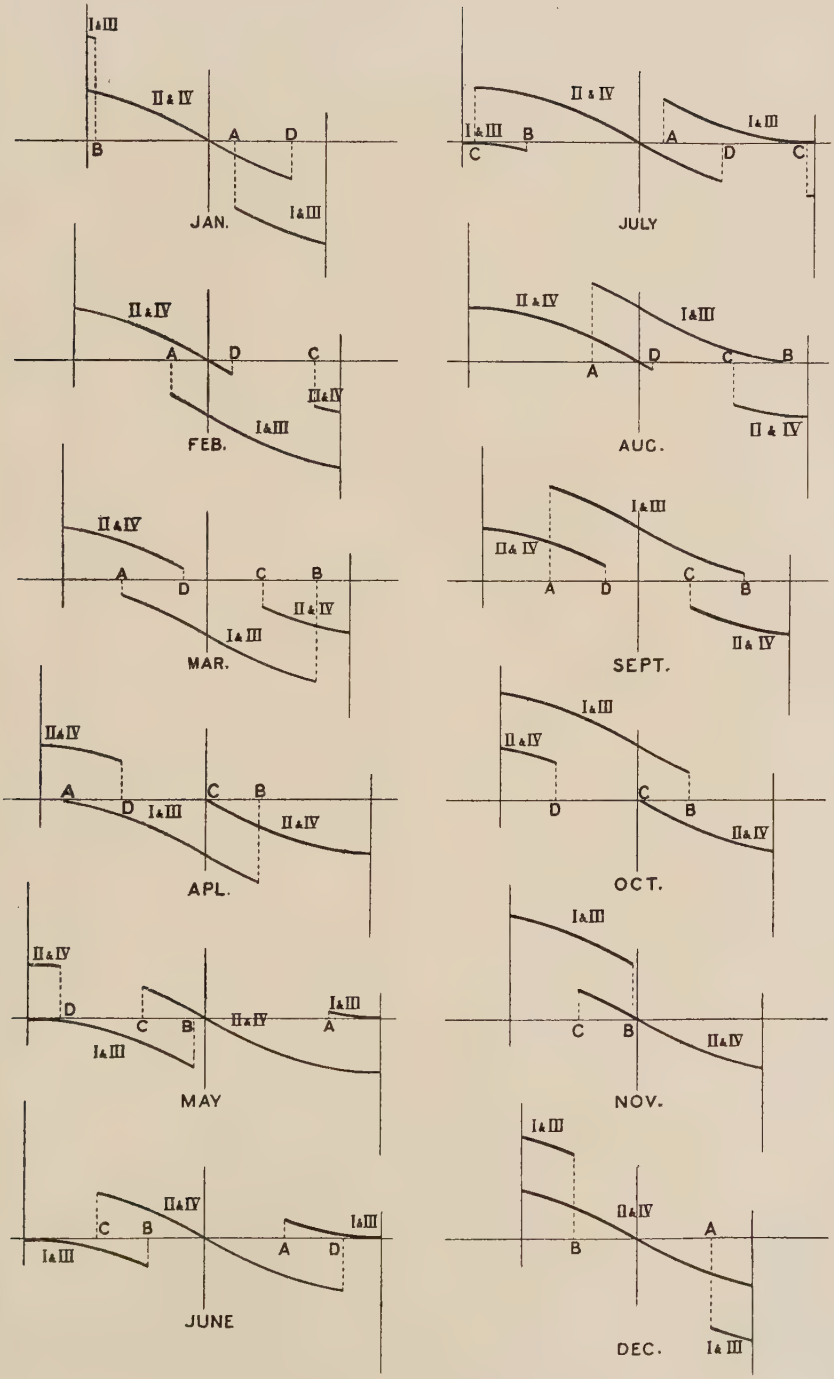
The essential difference between the proposed method and the classical methods employed at Pulkovo consists in the use of a continuous sequence of stars following one another at short intervals throughout the twenty-four hours of right ascension, instead of restricting the observations to a limited star list.

In the investigation dealt with in the first part of this paper, a considerable number of observations were involved which added little or no weight to the ultimate derivation of the aberration constant, though they were effective in strengthening the determination of the star position. Further, owing to the absence of midnight observations of right ascension in the winter and relatively short duration of the watches in the summer, the extreme factors attained probably averaged only about half those attainable.

There should be little difficulty, provided the observing strength is concentrated on those observations most effective for the aberration problem, in arriving at a determination carrying about three or four times the weight of the former as the outcome of a single year's observations.

By extending the programme over a longer interval, complete provision could be made for the elimination of proper motion and precession effects.

The parallax factors in right ascension are zero at midnight and have their greatest values, but with opposite signs, near dusk and dawn. If, therefore, the observations used for comparison with the midnight observations of right ascension be symmetrically distributed between dusk and dawn, the tendency will be to eliminate any small residual effects due to parallax. The most effective observations of declination are always made near dusk and dawn when the parallax factors are insignificant.



Aberration Factors in Latitude.

The Motion of the Perihelion of Mercury deduced from the classical Theory of Relativity. By L. Silberstein, Ph.D., Lecturer in Natural Philosophy at the University of Rome.

It is well known that as early as 1845 Le Verrier found that the motion of the perihelion of Mercury, as derived from observations of transits, was greater by 38″ per century than it should be from the perturbation due to all the other planets of our system. A recent discussion of the subsequent investigations has shown the excess of motion to be about 5″ greater, viz., per century,

$$\delta\varpi = +42''{\cdot}9.$$

Equally well known are the attempts of Newcomb and of Seeliger to account for this excess of motion of Mercury's perihelion. A discussion of Seeliger's results (which, broadly speaking, were very satisfactory) and a modification of his treatment have been given by H. Jeffreys (*M.N.*, vol. lxxvii. p. 112). On the other hand, a great sensation has been recently produced among astronomers by the surprising circumstance that Einstein's newest "*generalised* theory of relativity" has yielded for the said excess just its full value, *i.e.*, in round figures, 43″. In fact, Einstein gives in his recent paper,* for the angle ϵ through which the elliptic orbit of a planet is turned, in the direction of motion, per period T, the formula

$$\epsilon = 24\pi^3 \frac{a^2}{T^2c^2(1-e^2)}. \quad . \quad . \quad . \quad (G)$$

where a, e stand for the major semi-axis and the eccentricity of the orbit, and c is the velocity of light in empty space. Substituting $a = 0{\cdot}3871 \,.\, 1{\cdot}49 \,.\, 10^6$ km., $e = 0{\cdot}206$, $T \doteqdot 87{\cdot}97$ days, $c = 3 \,.\, 10^5$ km./sec., the reader will find, for $\delta\varpi$ per century, 43″·1, which is the desired angle. The reason why I have denoted the above formula by (G) is, with all respect due to Einstein, that identically the same formula was given eighteen years earlier by Gerber,† whose investigation, entirely independent of any relativity ("old" or "new"), seems to have passed unobserved, most likely owing to its badly supported fundamental assumptions. To enter upon these latter would not answer the purposes of the present paper. It may, however, be interesting to notice that Gerber replaces Newton's potential $\frac{M}{r}$ by $\frac{M}{r\left(1-\frac{1}{c}\frac{dr}{dt}\right)^2}$, where c is the "velocity of propagation of the gravitation potential";

* *Annalen der Physik*, vol. xlix., 1916, pp. 769–822. See also Professor de Sitter's papers in *M.N.*, vol. lxxvi., 1916, p. 699, and vol. lxxvii. p. 155.

† P. Gerber, *Zeitschr. math. Phys.*, xliii., 1898, pp. 93–104. A short account of Gerber's theory is given in *Enc. d. math. Wiss.*, vol. V. 1, pp. 49–51; a still shorter, and very unfair, account is given by Herr E. Gehrcke in *Annalen der Physik*, li., 1916, pp. 122–124.

rejecting the third, and the higher, powers of dr/cdt, Gerber obtains for any (isolated) planet in its motion round the central body the above formula. It is historically interesting that Gerber does not identify c with the velocity of light, but determines its value from the observed excess of the secular motion of the perihelion of Mercury, and finds $c = 305500$ km./sec., *i.e.* "surprisingly near the light velocity." Thus, whatever his theory, the formula (G), accounting for *the full excess* of Mercury's perihelion motion, will appropriately be called *Gerber's formula.*

Now, to repeat it, Gerber has deduced his formula from an untenable theory, or at least from one which has not been based upon well-established general principles. Einstein, eighteen years later, but undoubtedly without knowing Gerber's formula, has rediscovered it by deducing it from his "generalised" theory of relativity, which, in its turn, is again very far from being well established. In fact, notwithstanding its broadness and mathematical elegance, it certainly offers many serious difficulties in its very foundations, while none of its predictions of new phenomena, as the deflection of a ray by the sun, have thus far been verified. And even the fact that Einstein's new theory gives Gerber's formula, and therefore the *full* excess of 43″ for Mercury, does not seem to be decisive in its favour. As far as I can understand from Jeffreys's investigation,* it would rather alleviate the astronomer's difficulties if the Sun by itself gave only a *part* of these 43 seconds.

Under such circumstances it has seemed worth while to investigate, in as general a way as possible, how much of that excess will be accounted for by the ordinary or, as Einstein now calls it, the "old" theory of relativity, retaining the *constancy* of the velocity of light, and, of course, its independence of any gravitational field. It will be remembered that this "old" theory is not only unobjectionable in its foundations and considerably simpler than the "new" theory of relativity, but that it has also the physical advantage of accounting for a good number of important facts: it harmonises with the null-effects of Michelson-Morley's and of Trouton-Noble's experiments, it stands the test of experience in the domain of the best observations on β-particles, and it represents Fresnel's dragging coefficient in the most direct way, viz. as a simple corollary of kinematics.

So much to justify a renewal of the investigation of planetary motion on the basis of the "old" theory of relativity, as laid down in Einstein's famous paper of 1905, and developed formally by Minkowski two years later. The planetary motion has been treated by Minkowski himself in an appendix to his memoir (*Grundgleichungen, etc.*, of 1907 Dec. 21), in which, however, he limits himself to a short sketch and concludes by saying that the proposed relativistic modification would hardly show any observable deviations from the results of ordinary Newtonian mechanics. In

* *Loc. cit.*, see especially p. 113, and the final paragraph of the paper, p. 118.

fact, his law of gravitation would correspond in our notation (*cf. infra*) to $N = M_0 m_0/\gamma r^2$, and this gives, for a small planet (m_0/M_0 negligible),

$$\epsilon = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (M)$$

In more recent times the problem of planetary motion has been treated relativistically by Dr. Wacker (*Dissertation*, Tübingen, 1909), whose assumptions amount to adopting the law of force $N = M_0 m_0/r^2$, leading to

$$\epsilon = \tfrac{1}{6}(\mathrm{U}), \text{ } i.e. \text{ } \delta\varpi = 7''{\cdot}2 \text{ for } \mathrm{U} \quad . \quad . \quad . \quad (W)$$

in words, to $\frac{1}{6}$ of Gerber's value quoted at the outset.* This is at the same time just $\frac{1}{6}$ of the full observed excess for Mercury. Now, there is nothing particular about these two special laws. We shall see that the problem can be treated more broadly, and so as to account for the full observed excess. In our opinion the retouching of Newton's gravitation law must be guided by experience (observation) and not by guesswork or predilection for particular integers. At the same time we shall use the opportunity of simplifying the mathematical treatment of the corresponding equation of motion, which is usually, and without any intrinsic reasons, being made formidably complicated.

Let $\mathbf{r}$ be the vector (x, y, z) drawn from the Sun to the planet, a system attached to the former being our reference system S. Let t be the S-time, and $\mathbf{v} = d\mathbf{r}/dt$ the (ordinary) vector velocity of the planet with respect to S. Write

$$\gamma = (1 - \beta^2)^{-\frac{1}{2}}, \ \beta = v/c \quad . \quad . \quad . \quad (1)$$

where c is the velocity of light, assumed to be constant and invariant, of course. Let m_0 be the *rest-mass* of the planet, a constant by assumption, and

$$m = m_0\gamma \quad . \quad . \quad . \quad . \quad (2)$$

Finally, let $\mathbf{N}$, an ordinary three-dimensional vector (like $\mathbf{r}$ or $\mathbf{v}$) be the "Newtonian force" acting upon the planet. Then the equation of motion of the planet will be †

$$\frac{d}{dt}(m\mathbf{v}) \equiv \frac{d}{dt}\left(m\frac{d\mathbf{r}}{dt}\right) = \mathbf{N} \quad . \quad . \quad . \quad (3)$$

This can be read: The rate of increase of momentum, $m\mathbf{v}$, is equal to the force $\mathbf{N}$. The only departure from Newtonian mechanics is that the inertia-coefficient m is not a constant but a function of velocity, precisely as the familiar transversal mass of an electron.

* In revising the MS., I learn from Professor Eddington that de Sitter obtains, in *M.N.*, vol. lxxi., 1911, on Minkowski's and Lorentz's law of force, for Mercury, $\delta\tilde{\omega} = 0$ and $\delta\tilde{\omega} = 7''{\cdot}15$ respectively, which are evidently Minkowski's and Wacker's results.

† The reader unacquainted with the "old" theory may consult the author's *Theory of Relativity*, Macmillan, 1914, pp. 192 *et seq.*, where m is identical with the present m_0.

If the reader does not like even the "old" relativity, but is progressive enough to admit the inertia of energy, he may write $m = m_0 + \frac{1}{2}m_0\frac{v^2}{c^2}$, or $m = m_0(1 + \frac{1}{2}\beta^2)$, which to all purposes is the same thing as $m_0\gamma$. In fact, we shall ultimately retain but the second power of β, rejecting all higher powers of this exceedingly small fraction. Thus (3) as an equation of motion has nothing unfamiliar about it.

Next, without yet specifying the dependence of the force upon distance, let $\mathbf{N}$ be a central force, say,

$$\mathbf{N} = -N\mathbf{u}, \quad . \quad . \quad . \quad . \quad (4)$$

where $\mathbf{u}$ is a radial unit vector drawn away from the Sun, and N the absolute value of the force. With this symbol we shall have also $\mathbf{r} = r\mathbf{u}$. Under these circumstances the vector product $V\mathbf{rN}$ will vanish, and therefore by (3)

$$0 = V\mathbf{r}\frac{d}{dt}(m\mathbf{v}) = \frac{d}{dt}(mV\mathbf{rv}).$$

Thus

$$mV\mathbf{rv} = \mathbf{k}$$

will be a constant vector both in size and in direction, relative to the system of reference S, of course. The planet moves in a plane normal to $\mathbf{k}$, as it would according to Newtonian mechanics. The only difference is that m is slightly greater than its constant rest-value m_0. In other words, the *moment of momentum*, $\mathbf{k}$, is a constant vector, as with Newton, but the inertial coefficient m is slightly variable.* Writing k for the absolute value of $\mathbf{k}$, we have, in terms of the usual polar co-ordinates, r, θ,

$$mr^2\frac{d\theta}{dt} = k \quad . \quad . \quad . \quad . \quad (5)$$

Using this equation in (3), we have, by (4),

$$-N\mathbf{u} = \frac{k}{mr^2}\frac{d}{d\theta}\left(m\frac{d\mathbf{r}}{dt}\right) = \frac{k^2}{mr^2}\frac{d}{d\theta}\left(\frac{1}{r^2}\frac{d\mathbf{r}}{d\theta}\right).$$

Now, remembering that $\mathbf{r} = r\mathbf{u}$, the reader will easily verify the identity

$$\frac{d}{d\theta}\left(\frac{1}{r^2}\frac{d\mathbf{r}}{d\theta}\right) = -\left(\frac{d^2\rho}{d\theta^2} + \rho\right)\mathbf{u},$$

where $\rho = \frac{1}{r}$. Thus the equation of motion gives the scalar differential equation for the orbit,

$$\frac{d^2\rho}{d\theta^2} + \rho = \frac{mr^2}{k^2}N, \ \rho = \frac{1}{r} \quad . \quad . \quad . \quad (6)$$

where N is the absolute value of the central attractive force.

* These somewhat lengthy remarks are intended for those readers who are not at all familiar with relativity.

Thus far we have only assumed that **N** is a central force, N**u**, without specialising its absolute value or intensity N. Now, according to Newton's gravitation law we should have, in appropriate units, $N = M_0 m_0 / r^2$, where M_0 is the constant and, as we shall assume, relativistically invariant *rest-mass* * of the Sun. Instead of this let us take

$$N = \frac{M_0 m_0}{r^2} f(\gamma) \quad . \quad . \quad . \quad . \quad (7')$$

where f is an *arbitrary function* of $\gamma = (1 - \beta^2)^{-\frac{1}{2}}$, to be determined from experience, *i.e.* from astronomical observations. To begin with, we know only that, apart from second-order effects, Kepler's three laws, and therefore Newton's law of force, have to reappear. Thus we shall require $f(1) = 1$, and since the higher powers of β^2 are ultimately rejected, we can without any loss to generality write $f(\gamma) = \gamma^{n-1}$, that is,

$$N = \frac{M_0 m_0}{r^2} \gamma^{n-1} \quad . \quad . \quad . \quad . \quad (7)$$

where n is *any real number*. Its value (or the form of f) will be determined from the observed excesses of the motion of perihelia, which are second-order effects in the precise, technical sense of the word. Meanwhile we shall leave n undetermined.

As to relativistic requirements, the vector $\gamma \mathbf{N} = -\gamma N \mathbf{u}$ as given by (7′) or (7) is the space-part of a genuine four-vector or physical quaternion, as it should be. Thus the equation of motion (3), with (7), satisfies the Principle of Relativity. (For proof of the above statement see Note at the end of the paper.)

Introducing the law (7) into (6), we have, as the equation to the planetary orbit,

$$\frac{d^2\rho}{d\theta^2} + \rho = \frac{M_0 m_0^2}{k^2} \gamma^n \doteq \frac{M_0 m_0^2}{k^2} \left(1 + \frac{n}{2}\beta^2\right) \quad . \quad . \quad (8)$$

Next, multiplying the original equation of motion by **v**, scalarly, we have the equation of energy

$$c^2 m_0 \frac{d\gamma}{dt} = (\mathbf{N}\mathbf{v}) = M_0 m_0 \gamma^{n-1} \frac{d\rho}{dt},$$

whence, for any $n \neq 2$,† and denoting by E the integration constant, which is the total energy,

$$\gamma^{2-n} = (2 - n)\left(\frac{M_0 \rho}{c^2} + \frac{E}{m_0 c^2}\right),$$

* It will be noted that rest-masses may, but need not necessarily, be relativistic invariants. See, for instance, pp. 247 *et seq.* of my *Theory of Relativity*.

† And if $n = 2$, then $\log \gamma = \frac{M_0 \rho}{c^2} + \frac{E}{m_0 c^2}$, and, up to β^4, $\frac{1}{2}\beta^2 = \frac{M_0 \rho}{c^2} + \frac{E}{m_0 c^2}$, as in (9), apart from the meaning of the constant term.

or, up to β^4, and including the constant additive term $\frac{1}{n-2}$ in the energy constant,

$$\tfrac{1}{2}\beta^2 = \frac{M_0\rho}{c^2} + \frac{E}{m_0c^2} \quad . \quad . \quad . \quad . \quad (9)$$

Introducing this value into (8) and writing

$$g^2 = 1 - n\left(\frac{M_0m_0}{kc}\right)^2, \qquad C = \frac{M_0m_0^2}{g^2k^2}\left(1 + \frac{nE}{m_0c^2}\right) \quad . \quad (10a)$$

(and zero instead of the last term in the case $n = 2$), we have the differential equation of the orbit

$$\frac{d^2\rho}{d\theta^2} + g^2\rho = Cg^2 \quad . \quad . \quad . \quad . \quad (10)$$

where $\rho = 1/r$. Thus if θ is reckoned from the perihelion

$$\frac{1}{r} = \left(\frac{1}{r_0} - C\right)\cos g\theta + C,$$

whence also, for $\theta = \frac{\pi}{2}$, $1/C = r_{\pi/2} = a(1 - e^2)$, while $r_0 = a(1 - e)$.

Finally, therefore, the orbit becomes

$$\frac{1}{r} = \frac{1 + e\cos g\theta}{a(1 - e^2)} \quad . \quad . \quad . \quad . \quad (11)$$

i.e., if $e < 1$, *an ellipse with moving perihelion.* At the same time we have, by (10*a*), and since $C = 1 : r_{\pi/2}$,

$$\frac{1}{a(1 - e^2)} = \left(1 + \frac{nE}{m_0c^2}\right)\frac{M_0m_0^2}{g^2k^2} \quad . \quad . \quad . \quad (11a)$$

The angle ϵ through which the perihelion moves (or the orbit is turned round) in the direction of motion, *per period* of revolution, is, by the first of (10*a*),

$$\epsilon = 2\pi\left(\frac{1}{g} - 1\right) = 2\pi\left[1 - n\left(\frac{M_0m_0}{kc}\right)^2\right]^{-\frac{1}{2}} - 2\pi;$$

that is, with the rejection of the fourth power of the small fraction M_0m_0/kc,

$$\epsilon = n\pi\left(\frac{M_0m_0}{kc}\right)^2.$$

Here we can write, with more than sufficient accuracy, by (5), $k = mr^2\dot{\theta} \doteqdot m_0r^2\dot{\theta} = 2\pi abm_0/T$; and, by (11*a*), $M_0 = 4\pi^2a^3/T^2$, where T is the period of revolution and $b = a\sqrt{1 - e^2}$, the minor semi-axis.

Ultimately, therefore, the motion of the perihelion, per period, becomes

$$\epsilon = 4n\pi^3\frac{a^2}{T^2c^2(1 - e^2)}.* \quad . \quad . \quad . \quad (12)$$

* The two cases treated by Minkowski and by Dr. Wacker, as mentioned before, correspond to $n = 0$ and $n = 1$ respectively. Gerber's formula (G) follows for $n = 6$.

Thus the law of force (7),

$$N = M_0 \frac{m_0}{r^2}\gamma^{n-1} = M_0 \frac{m}{r^2}\gamma^{n-2},$$

or its four-dimensional equivalent, satisfying the Principle of Relativity, gives the motion of perihelion (12), where n is *any* real number. The *numerical value* of n, it would seem, *is to be decided upon the basis of astronomical observations.* If it is true that Mercury's excess of 43″ per century has to be thrown *entirely* upon the Sun, then make

$$n = +6;$$

and if only in part, then make it less than six. There is, however, a strong presumption that the whole excess is to be attributed to the Sun's gravitation. If so, then n, determined from Mercury's $\delta\varpi$, is just (or very nearly) *six*, and gives for Venus the value $\delta\varpi = 8''{\cdot}6$. The required law of force would then be

$$N = M_0 \frac{m_0}{r^2}\gamma^5 = M_0 \frac{m}{r^2}\gamma^4 \quad . \quad . \quad . \quad (7a)$$

where $m = m_0\gamma \doteqdot m_0\left(1 + \frac{1}{2}\frac{v^2}{c^2}\right)$ is, according to the well-established proportionality of heavy and inert mass, the *heavy* mass of the planet. Why n is just six, or the exponent of γ in the latter formula just four, I am sure I do not know. But as little do we know "why" the exponent of r is "just" or exceedingly nearly equal -2.

Such a naturalistic method of improving Newton's law of gravitation seems a great deal safer than those based on fantastic constructions or rash generalisations. Since the capital part of that great law, the value of the exponent of r, has (through Kepler's empirical laws) been ultimately derived from and solely supported by experience, viz. by the observations on the shape of the orbits, it seems but natural that the exponent of a retouching factor, such as γ, should be determined from refined experience, viz. from the observed true excess of the secular rotation of the axes of those orbits. At any rate, the "old" Principle of Relativity has turned out to be broad enough to give us a (very simple and) sufficiently general form of the retouching factor of Newton's law, as required for the said secular excesses.

London:
1917 *March.*

Note to page 507.—In order to see that the law of force (7′), and therefore also (7), satisfies the Principle of Relativity, let us return to the general equation of motion (3). If τ be the proper time of the planet, then $d/dt = d/\gamma d\tau$, so that (3) becomes

$$\gamma\mathbf{N} = m_0\frac{d}{d\tau}(\gamma\mathbf{v}) = m_0\frac{d^2\mathbf{r}}{d\tau^2}.$$

Now, m_0 is, by assumption, a relativistic invariant, and $d^2\mathbf{r}/d\tau^2$ is the space-part of a relativistic four-vector or (as I call it) of a physical quaternion, viz. $d^2q/d\tau^2$, where $q=\mathbf{r}+ict$; and τ, as is well known, is an invariant. Thus the relativistic requirement is that $\gamma\mathbf{N}$ (not $\mathbf{N}$ itself) *should be the space-part of a four-vector* (or the vector of a physical quaternion) X, viz.

$$\gamma\left[\mathbf{N}+\frac{i}{c}(\mathbf{N}\mathbf{v})\right]=\mathrm{X}=\frac{d^2q}{d\tau^2}\quad .\quad .\quad .\quad (13)$$

Now, let $\mathrm{Y}=dq/d\tau$ be the four-velocity of the planet, and similarly $\mathrm{Y}_s=dq_s/d\tau_s$ that of the Sun, and write $\mathrm{R}=q-q_s=\mathbf{r}-\mathbf{r}_s+ic(t-t_s)$, so that Y, Y_s, R are all four-vectors. With these symbols, and denoting by ( ) the "scalar product" of two four-vectors, write

$$\mathrm{X}=\frac{\mathrm{F}(\mathrm{Y}\mathrm{Y}_s)}{(\mathrm{R}\mathrm{Y}_s)^3}\{(\mathrm{Y}_s\mathrm{Y})\mathrm{R}-(\mathrm{Y}\mathrm{R})\mathrm{Y}_s\}\quad .\quad .\quad (14)$$

where F is any function of $(\mathrm{Y}\mathrm{Y}_s)$. Then, the bracketed expression being a four-vector, and its scalar coefficient even an invariant, X will be a genuine four-vector as required. In fact, the above X is but a large sub-case of the four-vectors constructed by Poincaré (*Rend. del Circolo Mat. di Palermo*, xxi., fasc. i., 1906). In comparing with his memoir notice that Poincaré's C is our $-\frac{1}{c^2}(\mathrm{Y}\mathrm{Y}_s)$. Now, the space-part of (14) is precisely of the form of the law of force (7′). To see this, remember that our reference system S has been the rest-system of the attracting centre (the Sun), and this will be a legitimate system provided that m_0/M_0 is small.* This means that $\mathbf{r}_s=0$, $\mathbf{v}_s=0$, and therefore $\mathrm{Y}_s=ic\gamma_s=ic$, and $c(t-t_s)=-r$, *i.e.* $\mathrm{R}=\mathbf{r}-i\mathbf{r}$. Under these circumstances we have

$$(\mathrm{Y}\mathrm{Y}_s)=-c^2\gamma,\quad (\mathrm{R}\mathrm{Y}_s)=cr,$$

and therefore for $\gamma\mathbf{N}$, or the vector of the quaternion (14),

$$\gamma\mathbf{N}=\mathrm{F}(-c^2\gamma)\,.\,\frac{\mathbf{r}}{c^3r^3}=\frac{\mathrm{F}(-c^2\gamma)}{c^3}\,\frac{\mathbf{u}}{r^2},$$

or N equal an arbitrary function of γ divided by r^2, as in formula (7′), of which the adopted law of force (7) is but a particular case.

* In the case of Mercury this is at least as small as 1 : 5,600,000, and, according to Newcomb, even as 10^{-7}. It is also sufficiently small for Venus, etc. For a double star, of course, (14) itself would have to be used, without the simplification which has been possible in our case.

The Third Line Spectrum of Oxygen. By A. Fowler, F.R.S., and J. Brooksbank, B.Sc., D.I.C. (Plate 6.)

One of the most interesting problems of experimental astrophysics is that of the reproduction of lines which occur in the spectra of stars of early type, and in nebulæ. Assuming the identity of celestial and terrestrial matter, the method of experiment would seem to be clearly indicated by the important results which have previously been obtained in this connection. The investigations of silicon, carbon, nitrogen, and sulphur, carried out by Lockyer and his assistants,* have established that in the case of these elements the spectrum can be successively changed by increasing the energy of excitation, and that the lines which appear at the different stages in the laboratory experiments appear with greatest intensity at successively higher stages of the stellar sequence. Several lines of the early B stars were identified by Lockyer in this way with lines produced by relatively strong discharges in the case of the elements above named. Subsequent work on helium † showed that a modified spectrum of this gas, also produced by very strong discharges, accounted for the line 4686 and the lines of the Pickering series, which occur not only in the earliest B stars, but also in Wolf-Rayet stars and gaseous nebulæ. The more recent work of Merton ‡ similarly revealed a modification of the line spectrum of carbon beyond that observed by Lockyer, and showed that these lines were probably identical with some of the most characteristic lines in the spectra of Wolf-Rayet stars.

It has seemed to us important to extend such observations to as many substances as possible, with a view to the identification of additional stellar lines, and to enlarge the range of spectrum observed for the purpose of investigating the constitution of the spectra produced at the various stages. One of the substances we have studied is oxygen, and although but few of the lines developed under the most intense discharges fall within the range of spectra which can be observed in celestial bodies, it may be useful to present a brief account of the results obtained, in so far as they may be of use in connection with stellar spectra.

Two distinct line spectra of oxygen were already well known, under the names "compound line" and "elementary line" spectra,

* *Proc. Roy. Soc.*, vol. lxxiv. p. 296 (1904); vol. lxxxii. p. 532 (1909), and other papers.

† A. Fowler, *M.N.*, vol. lxxiii. p. 62 (1912); *Phil. Trans.*, A, vol. ccxiv. p. 254 (1914).

‡ *Proc. Roy. Soc.*, A, vol. xci. p. 498 (1915).

assigned by Schuster in 1879.* These spectra are produced respectively by uncondensed and comparatively weak condensed discharges, and the names given by Schuster were based upon the supposition that complex and simplified molecules were involved in the two cases. In connection with their discovery that the lines of the compound spectrum could be arranged in well-defined series, Runge and Paschen † proposed to call this the "series spectrum" of oxygen, but the name does not seem particularly appropriate, as it is by no means certain that the elementary line spectrum will not also be found to include series lines.

The lines of oxygen which were shown to be present in the solar spectrum by Runge and Paschen ‡ are the components of a narrow triplet in the extreme red (near λ 7770) which forms part of the compound line spectrum. It was the elementary line spectrum, requiring greater energy for its production, which was discovered in some of the B stars by McClean in 1897,§ and subsequently confirmed by Gill and by Lockyer.

By analogy with metallic spectra the compound line and the elementary line spectra would appear to correspond with arc and spark spectra respectivelv, so that the elementary line spectrum might properly be called the spark or enhanced line spectrum of oxygen. Such a designation would be in entire agreement with the occurrence of the elementary line spectrum in early type stars.

The probability of a further modification of the line spectrum of oxygen was first suggested by the early observations of Schuster and Roscoe,‖ who found that two lines in the green and greenish yellow "only appeared at very high temperatures,"—that is, when a condensed discharge was passed through the gas in a short and narrow capillary tube. The wave-length of one of these lines was given as 5591, which may be put at 5592 on the scale of Rowland; the position of the second line was not stated, and we have not identified it with certainty. The line at λ 5592 is apparently identical with one of those assigned to carbon by Merton, but our experiments agree with those of Schuster and Roscoe in showing it to be due to oxygen. It appears with undiminished strength as the oxygen is purified, and persists when all known lines of carbon, including those discovered by Merton in association with λ 5592, have disappeared. It is probable that in Merton's experiment the line was derived from the oxygen of carbon dioxide.

In 1906 additional lines of the type of 5592 were observed by Lunt, though he was apparently unacquainted with the above-mentioned observation by Schuster and Roscoe. These lines—in

* *Phil. Trans.*, vol. clxx. p. 41 (1879).
† *Astrophys. Journ.*, vol. viii. p. 70 (1898).
‡ *Ibid.*, vol. iv. p. 317 (1896).
§ *Proc. Roy. Soc.*, vol. lxi. p. 215 ; vol. lxii. p. 418.
‖ *Mem. Manchester Lit. and Philos. Soc.*, 3rd series, vol. vii. p. 82 (1880).

the near ultra-violet—were found by Lunt to be brought out by reducing the pressure of the gas, or by widening the air-gap in the secondary circuit when the pressure of the gas was such as would not otherwise have given rise to them. The brightest of them were at wave-lengths 3754·9, 3757·4, 3760·0, and 3961·8.* Lunt concluded that these lines were probably due to oxygen, but, in view of their peculiar behaviour, the possibility of their being due to an impurity was not overlooked.

Our observations are in entire agreement with those of Lunt, and we have found no reason to suppose that any of the lines in question are due to substances other than oxygen. These lines, in fact, only appear with any considerable strength under the action of very strong discharges, and with greater facility when the pressure of the gas is low; they increase in relative importance as the intensity of the discharge becomes greater. It would seem that they form part of a third line spectrum, which may be distinguished, in general terms, as the "super-spark" spectrum of oxygen. On the plan adopted by Lockyer in the case of the four groups of lines of silicon, we propose to designate the compound line, the elementary line, and the third line spectra as O. I., O. II., and O. III., corresponding respectively to the arc, spark, and super-spark stages. As in the case of other elements, the third spectrum might be expected to occur without the second under favourable conditions, such as may possibly be found in some of the stars.

Our observations of the oxygen spectra have extended over the range λ 7000 to λ 2000, and they show that although lines of O. III. are very numerous in the ultra-violet, there are no conspicuous members of this class between Schuster and Roscoe's λ 5592 and Lunt's 3961·8. The chief lines of O. III. between λ 7000 and λ 3700 are given in Table I., which also includes the brighter lines of O. II. in the near ultra-violet in order to indicate the difference of behaviour. The first two columns give the wave-lengths, on Rowland's scale, of lines belonging to the two groups, while the third and fourth show the variations in the intensities of the lines in passing from moderate to very strong discharges. Lunt's wave-lengths and intensities for corresponding lines are also given for comparison; this column includes all the lines of O. III. given by Lunt, but only the brighter lines of O. II. which are tabulated by him.

It will be observed that Lunt's spectrum is intermediate between the two given under O. II. and O. III. Any intermediate stage can, in fact, readily be obtained by suitable adjustment of the gas pressure, diameter of capillary tube, and intensity of discharge (see Plate 6).

* *Annals of the Cape Observatory*, vol. x., part ii., p. 26B (1906).

TABLE I.

The Chief Lines of Oxygen III. in the Region λ 7000 – λ 3700, *and of Oxygen II. in the Region* λ 4000 – λ 3700.

Oxygen II.	Oxygen III.	Intensity.		Lunt.	
		Moderate Discharge.	Strong Discharge.	λ.	Intensity.
	5592·57	...	6	Out of range	
Many lines					
3982·87	...	5	5	3982·93	6 *
3973·41	...	10	10	3973·51	9 *
	3961·75	...	6	3961·82	3
3954·52	...	7	7	3954·64	7 *
3945·18	...	5	5	3945·26	6 *
3919·48	...	5	5	3919·51	6 *
3912·10	...	8	8	3912·19	7 *
3882·34	...	6	6	3882·37	6 *
	3791·47	...	5	3791·46	1
	3774·23	...	5	3774·22	1
	3760·09	...	8	3760·04	6
	3757·40	...	4	3757·42	3
	3754·86	...	6	3754·88	5
3749·71	...	10	10	3749·63	7 *
3740·10	...	5	5	3739·95	0
3727·55	...	9	9	3727·49	6 *
	3715·25	...	4	3715·23	0
3712·80	...	8	8	3712·87	5 *
	3707·41	...	3		
	3703·50	...	2	3703·53	1
	3702·95	...	2		
	3698·86	...	1–2	3698·87	0

Occurrence of O. III. Lines in Stellar Spectra.

In view of the relative behaviour of the different groups of lines in the laboratory, it might be expected that lines of O. III. would appear in the spectra of B stars of somewhat earlier type than those which exhibit the lines of O. II. The stellar observations are less consistent and complete than might be desired for the purpose of this inquiry, chiefly because the strongest lines of O. III. lie outside the region to which most attention has been given in the observations of stellar spectra. Photographic records in the green and yellow regions of stellar spectra are unfortunately

* Observed by Lunt in stellar spectra taken at the Cape Observatory, chiefly in β Crucis and ε Canis majoris, which are both of type B1.

very few, and published wave-lengths for the ultra-violet beyond K appear to have been based chiefly on plates obtained with small dispersion. Nevertheless, the evidence for the occurrence of O. III. lines in stars is fairly consistent, as may be gathered from the data given in Tables II. and III. Table II. includes all the lines of O. III. of intensity 5 or greater in the region λ 7000 to λ 3700, and for purposes of comparison Table III. shows the behaviour of some of the chief lines of O. II. in the region λ 4000 to λ 3900.

Details of the stellar spectra are taken from the South Kensington * and Harvard † publications. Lockyer and Baxandall's typical stars are γ Orionis (B2) and ε Orionis (Bo). Those of Harvard are σ Sagittarii (B3), γ Orionis (B2), β Centauri (B1), and ε Orionis (Bo) for all the lines except 5592, in which case the stars are λ Scorpii (B2), β Crucis (B1), and ε Orionis (Bo). In the Harvard tables "o" indicates that a satisfactory negative failed to show the line, while ". ." means that the line was possibly obscured by some other line, or may have been missed through imperfect focus or other causes.

For details of the Wolf-Rayet star B.D. + 30° 3639 we are indebted to Dr. W. H. Wright, of the Lick Observatory, who has very kindly placed an unpublished table of wave-lengths at our disposal.

TABLE II.

Lines of Oxygen III. in Stellar Spectra.

Oxygen III.		Stars, Lockyer and Baxandall.			Stars, Harvard.					B.D. +30° 3639. Wright.
λ.	Intensity.	λ.	B2.	Bo.	λ.	B3.	B2.	B1.	Bo.	λ.
5592·57	6	Out of range			5588	o	o	o	2	5593
3961·75	6	3961·8	1–2	2–3	3962	o	o	o	2	3962·2
3791·47	5	Out of range			3790·9	o	..	..	2	3793·6±
3774·23	5	,,								
3760·09	8	,,			3760·1	o	..	5	5	3760±
3754·86	6	,,								

TABLE III.

Lines of Oxygen II. in Stellar Spectra.

Oxygen II.		Stars, Lockyer and Baxandall.			Stars, Harvard.					B.D. +30° 3639. Wright.
λ.	Intensity.	λ.	B2.	Bo.	λ.	B3.	B2.	B1.	Bo.	
3982 87	5	3982·9	1–2	1	3982·8	o	1	2	2	Not seen
3973·41	10	3973·4	2	1–2						,,
3954·52	7	3954·6	1–2	2	3954·8	1	2	2	2	,,
3945·18	5	3945·3	1–2	1–2	3945·0	1	2	2	2	,,
3912·10	8	3912·2	2		3912·2	1	3	3	2	,,

The line 5592 is well known in the spectra of Wolf-Rayet stars, and Wright's wave-length is in sufficient agreement with that determined in the laboratory to make the identification with O. III. very probable. The Harvard wave-length 5588 is rather low, but

* *Publications of the Solar Physics Committee*, 1914.

† *Annals H.C.O.*, vol. xxviii., part ii. pp. 235, 237 (1901).

the behaviour of the line in the stars corresponds precisely with that of other O. III. lines, and it may reasonably be supposed to be identical with 5592.

The presence of λ 3961·75 in stars appears to be well established by agreement in wave-length, though there is a notable discordance between the two sets of stellar data as regards the occurrence of this line in γ Orionis (B2); possibly the Harvard dispersion was insufficient to separate it from the adjacent helium line λ 3964·9.

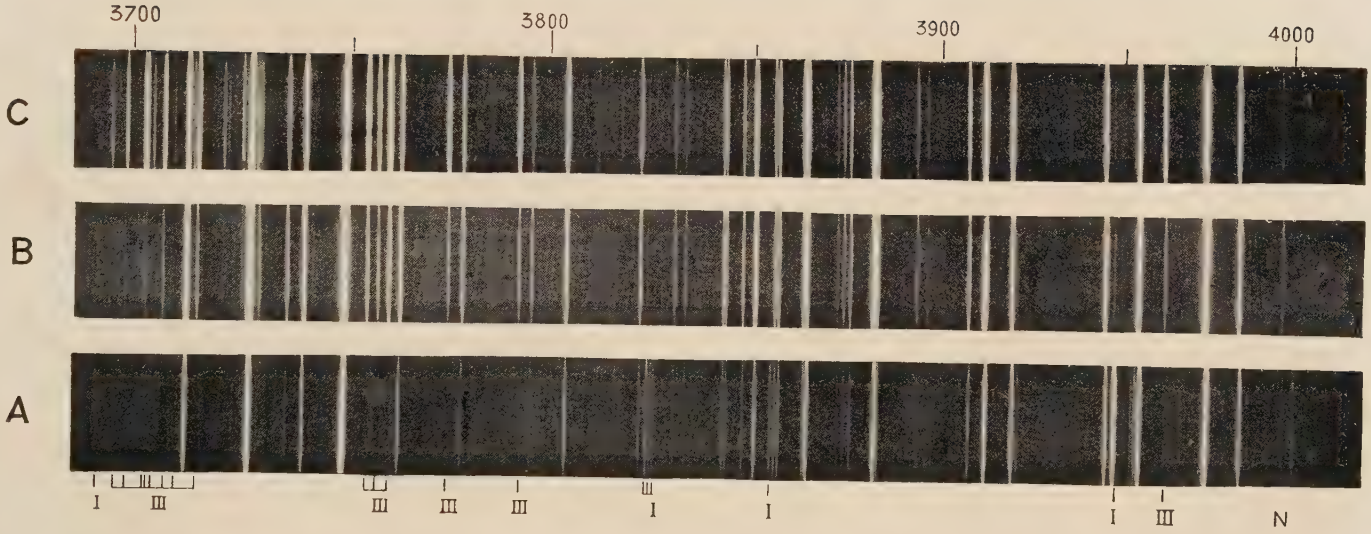
The line λ 3791·47 corresponds closely with the stellar line 3790·9 and with the Wolf-Rayet line λ 3793·6 ±. The strongest of the O. III. lines, λ 3760·09, is apparently represented in the Wolf-Rayet star, and also by a comparatively strong line in the B1 and Bo stars; the occurrence of this line in the B1 star, and its high intensity both in B1 and Bo, however, suggest that a line of some other element may possibly also be involved. Dr. Wright also points out that a line about this position occurs in the planetary nebula N.G.C. 7027, where no line in the position of λ 5592 has yet been photographed.

Two of the brightest O. III. lines, 3774·23 and 3754·86, do not appear in the Harvard lists, but it is possible that they may have escaped record on account of their proximity to the hydrogen lines 3770·8 and 3750·2; the Harvard plates have, in fact, similarly failed to reveal the strong O. II. line 3973·4, given by Lockyer and Baxandall, and by Lunt, apparently because it is too close to the hydrogen line 3970·2.

It will be seen that there is substantial evidence that the brighter lines of O. III. are present in some of the stars, and a comparison with Table III. shows that the circumstances of their appearance are distinctly different from those of the lines of the O. II. group. The tables indicate that while the latter first appear faintly in stars of type B3, and cease to be visible in the Wolf-Rayet star, the O. III. lines certainly do not appear earlier than type B2, and have their greatest development in type Bo, besides persisting to the Wolf-Rayet type. This difference in the behaviour of the two sets of lines in the stars is in strict accordance with the variations of the oxygen spectrum observed in the laboratory.

The stellar lines which are thus probably to be identified with the third line spectrum of oxygen are few in number, but it is interesting to have found in oxygen another example of the occurrence in stars of the earliest types of lines which only appear in experimental spectra when the strongest discharges are employed. It is of special interest to note that oxygen is now probably to be added to hydrogen, helium, and carbon as representing the few known elements which exhibit their spectra in stars of the Wolf-Rayet class. The importance of such identifications does not end with the recognition of the chemical elements which are involved; they may be of even greater significance as tending to prove that in the early-type stars, and probably also in nebulæ, we are presented with phenomena which result either from the effects of powerful electrical actions or of very elevated temperatures.

LINE SPECTRUM OF OXYGEN.



C. Strong condensed discharge.

B. Moderate condensed discharge.

A. Feeble condensed discharge.

Description of Plate.

The photographs reproduced have been enlarged from negatives obtained with a quartz spectrograph of the Littrow type, giving a linear dispersion of 10 A. per mm. in the neighbourhood of λ 3800. They represent the spectra produced in a capillary tube containing sensibly pure oxygen, by discharges from a 12-inch coil with varying capacity in the secondary circuit. In passing from A to B, and from B to C, the capacity was increased, and the pressure of the gas reduced.

A shows lines of O. I. and O. II., while those of O. III. are absent.

B shows the disappearance of lines of O. I., and the appearance of lines of O. III.

C shows the increased intensity of lines of O. III. as compared with those of O. II. when very strong discharges are passed through the gas at low pressures.

The chief lines may be identified by the scale of wave-lengths, and reference to Table I. The line at λ 3995 is due to a trace of nitrogen. In comparing intensities in the different spectra, it should be noted that spectrum A is rather underexposed in comparison with B and C.

Missing B.D. Stars. By J. G. Hagen, S.J.

The list of 81 B.D. stars within the zone +40° to +53° Dec., which were found "missing or faint" by the Rev. T. E. Espin (*M.N.*, Nov. 1916), forms a valuable addition to those published by Chandler, Pickering, and others. In view of their great number it seems rather remarkable that the writer met with only 28 cases of the kind in an area of 590 square degrees, which is that covered by the "Atlas of Var. Stars" (if we except Series V.). They may be found in the "Aggiunte al Catalogo dell' A.S.V.," *Spec. Vat.*, xi. (1916), 234, table xii*b*. Only one of them occurs also in Rev. T. E. Espin's list, No. 3 = +47° 192. The star is not really missing, for it is identical with +47° 193. The duplicate number had its origin in an erroneous Dec., and was suppressed by Schönfeld himself in *A.N.*, cxvi. p. 272.

This is a good illustration of "error in the original observation," one of the three sources to which Sir William Herschel reduces the missing stars (*Phil. Trans.*, lxxiii. [1783], 250–254).

"Variation," if periodical, may cause the missing of a B.D. star at intervals, but if progressive, it will cause the star to be missing for ever. This latter case, however, is extremely rare. As an example may be adduced B.D. +16° 354, now Nova Arietis 1854. It is on the charts of Argelander and of Chacornac, near T Arietis, but was missed by Winnecke and Schönfeld and has never been seen again (*A.N.*, 4523 and 4579).

"Proper motion" may exceptionally shift a star as much as 5′ or more in half a century, and thus make it apparently disappear. As a rule, however, the shift will remain far within the errors of the B.D. In fact, not a single B.D. star was missed on account of P.M. on all the 290 charts of the Atlas (not counting Series V.).

What the writer wishes especially to mention from his own experience are two other kinds of errors that have caused spurious

stars in the B.D. One of them may be called *bad focus*, through which faint stars close together are observed as one bright star. In the Atlas no less than 70 B.D. stars are resolved into components. A striking example is $+39^{\circ}$ 4866, $9^{m}\cdot5$. It consists of three stars, the brightest of which (to the north) hardly reaches mag. 11·5. Variation is excluded, since none of them showed a change on the Harvard plates between 1891 and 1906, and the B.D. position is assured by two "finder zones" 1856 Aug. 30, Sept. 26. The star $9^{m}\cdot5$ was entered on the working chart S Lacertæ (III. 8068) at $+2^{m}\ 46^{s}\ +17'\cdot1$, but afterwards cancelled as not belonging there.

A rather curious error, which has apparently never been pointed out before, consists in the *combination of the R.A. of one star with the Dec. of another*. The spurious star then appears at the right angle of a triangle of which the real stars form the vertices of the acute angles. Six cases of this kind were found during the work on the Atlas: -21° 1062, $+7^{\circ}$ 1288, $+19^{\circ}$ 4513, $+32^{\circ}$ 3524, $+33^{\circ}$ 469, $+57^{\circ}$ 558, a rather small number, when we remember that the two co-ordinates were written down by different persons, one at the telescope and the other, beneath the floor, in front of the clock.

Vatican Observatory:
1917 *March* 6.

Observations made during the Partial Eclipse of the Sun on 1917 *January* 22 *at the temporary University Observatory, Rostow-on-Don.* By Professor S. D. Tscherny.

Phenomenon.	Rostow-on-Don. Mean Time. h m s	Remarks.
First contact	20 52 25·2	Good
Occultation of the following border of sun-spot 1 by the Moon	21 11 12·7	Good
First contact of the Moon and the large umbra of sun-spot 2	21 33 10·7	
Second contact of the Moon and the large umbra of sun-spot 2	21 33 31·7	
Occultation of the following border of sun-spot 3 by the Moon	21 35 37·7	
Occultation of the following border of sun-spot 4 by the Moon	21 37 21·7	
First contact of the Moon and the umbra of sun-spot 5	21 39 18·7	
Second contact of the Moon and the umbra of sun-spot 5	21 39 54·7	
First contact of the Moon and the large umbra of sun-spot 6	21 48 5·7	
Second contact of the Moon and the large umbra of sun-spot 6	21 48 39·7	
Last contact	23 8 45·7	Satisfactory. Through Str.-Cum.

Instruments used:—

(1) 30-mm. Bahrenfeld altazimuth, power 31. (First contact.)
(2) 71-mm. Vion refractor, power 71. (All other observations.)

The magnitude of the eclipse in Rostow-on-Don was 0·64.

Position of Observatory: latitude, 47° 13′ 9″·4 N.; longitude, $2^h\ 38^m\ 49^s{\cdot}48$ E. The comparison of the observed times with those calculated * for our Observatory gives—

	Observ.–Calcul.
First contact . .	$+3^s{\cdot}6$
Last contact . .	$-8^s{\cdot}9$

The following meteorological observations were made by B. Yankowich:—

Summary of Barometer and Thermometer Readings.

Local M.T.	Aneroid Barometer.	Thermometer.
h m		
20 42	759·2	−10°·0 C.
21 32	757·8	− 9·5 (max.)
22 0	757·6	− 9·8 (min.)
22 55	757·5	− 9·2
23 9	757·5	− 8·8

The amount of cloud varied from 2 to 5.

University Observatory,
Rostow-on-Don:
1917 *January* 25.

Note on the possible Attraction between Photographic Images.†
By H. H. Turner, D.Sc., F.R.S., Savilian Professor.

1. In 1910 a copy of a photograph ‡ taken at Oxford was shown to the Society, in which the image of a bright star had apparently caused a considerable distortion in a neighbouring réseau line (see *The Observatory*, vol. xxxiii. p. 467, 1910 Dec.). Such a disturbance is certainly not systematic, for we have scrutinised all the cases of stars brighter than 4·5 mag. (R.H.P.)

* "Elements of the Eclipse of the Sun," *Connaissance des Temps pour l'an 1917*, p. 594.

† Since this paper was put in type, attention has been drawn to the interesting note by W. W. Campbell in *L.O.B.*, No. 284, "On Systematic Errors in the Measured Interval between Close Lines."

‡ Plate 2712, R.A. $5^h\ 56^m$, Dec. +25°. The bright star is 139 Tauri, mag. 4·90 R.H.P.

on our Oxford plates without finding another instance. It may have been due to some accident; but the cause is still obscure, as in the case cited many years ago by Professor Pritchard of a deformation amounting to 1″·2 (*M.N.*, xlvi. p. 442). Still what may happen once or twice grossly may happen in minor degree at other times; and it seems desirable to watch the behaviour of closely contiguous images with some care.

2. What recalled to mind the instances cited was the following circumstance. The war having made it impossible to obtain réseaux from the former makers, the Director of the National Physical Laboratory very kindly undertook to rule one, and sent us a successful result some months ago, which has ultimately been despatched to India, where the need of it was greatest. But, to test its accuracy at Oxford, it was twice copied on to the same plate, turning it round through 90° between the exposures. Measurement of the distances apart of the two copies in various parts of the plate, while they do not give an absolute determination of error, are quite sufficient to indicate whether the errors are small: for quite different parts of the réseau are brought into juxtaposition, and moreover the x lines are compared with the y lines. But the difference of orientation was not quite 90°, so that one set of lines crossed the other at a small angle. The differences should decrease uniformly if the lines are straight: but it was found that in the neighbourhood of the crossing point they did not do so. The observed separation gradually became less than the theoretical, on both sides of the crossing point. There is no need to give details of so simple an operation: the general result of a good many measures was as follows:—

Separation in réseau intervals .	0·015	·020	·030	·040	·050	·060	·070
Mean error of separation in units of 0·0001 . .	+11	+13	+1	+7	+5	+2	0

Beyond a separation of 0·070 (or one-third of a millimetre, or 20″ in *Astrographic Catalogue* measure) the error was sensibly zero. From there it gradually increased to ·0012 (·006 of a millimetre, or 0″·36 for the *Astrographic Catalogue* measures) at the point where the images were practically in contact, within which measures were not made. After noticing the effect on the plate made to test the réseau, some other plates were made with longer exposures in order to see whether denser and thicker images gave more "attraction": but no sensible increase of the phenomenon was found, much to our surprise.

3. It seemed desirable to examine the measures of close double stars given in the *Oxford Astrographic Catalogue*, and Mr. Bellamy has done this, finding much the same order of error for distances less than 7″, though nothing sensible beyond this. His results are given in a separate note. It does not follow, of course, that there is any actual displacement of the photographic film. If we visualise two photographic images as similar to two probability curves, when the maxima are close together, the portion between

is the sum of the two ordinates: and it is easily seen that the maxima will tend to approach. But whatever the origin of the error, it should be noted and allowed for in measuring close images. Probably in most measures for stellar parallax the images are too far apart to be affected. But in measures of spectrum lines, is it not possible that we have here a source of error, for closely contiguous lines, which has not been entirely eliminated? Tests could be made by photographing a straight line (from a réseau say) crossing a spectrum at a slight inclination to the spectrum lines. It might show distortions of the kind above noted.

Systematic Errors in Photographic Distances of Double Stars.

By F. A. Bellamy, M.A.

The recent examination of a new réseau, by making two copies on the same plate, called attention to the fact that the crossing of two réseau lines at a slight inclination showed an interference of one image with the other; and it seemed probable that photographic measures of double stars might afford some information upon the point.

Some years ago I undertook the computation of the distances and position-angles of all pairs of stars—to a definite limit in distance — which are given in the seven volumes of the Oxford Astrographic Catalogue, zones $+31^\circ$ to $+25^\circ$. I specially selected those, in the first instance, which are given in Burnham's "General Catalogue of Double Stars," then recently published.

At that time two of the Oxford zones were not printed, and the investigation has remained in MS. awaiting time for its completion of the stars in zones $+25^\circ$ and $+27^\circ$. But when the question of the effect of one photographic image on another was raised, it was a simple matter to make use of some of this material to test this question.

Zone $+26^\circ$ was used in the first instance, but there being too few stars below the distance of $9''$, all the pairs in zones $+28^\circ$ to $+31^\circ$ were collected into groups and the results are tabulated in Table I., where the mean groups for Burnham's Δ are given in columns 1 and 2, $O-\beta$ in columns 3 and 4 are the differences of Oxford *minus* Burnham for distance and position-angle, r_1+r_2 is the half sum of the Oxford diameters (d) of the stars expressed in seconds of arc in column 5, and the last column gives the number of pairs used: altogether 436 pairs are included.

TABLE I.

Means of the Groups for $O-\beta$.

Group.	Mean. Δ.	$O-\beta$. Δ.	$O-\beta$. $\angle$.	Oxford. r_1+r_2.	No. Pairs.
3·0–3·9	3·7	−0·76	0·0	8·1	3
4·0–4·9	4·5	0·46	+0·3	5·1	9
5·0–5·9	5·5	−0·38	+0·2	6·6	9
6·0–6·9	6·5	−0·14	−0·5	7·2	29
7·0–7·9	7·6	−0·15	0·0	6·6	20
8·0–8·9	8·5	−0·06	−1·2	7·5	19
9·0–14·9	11·9	+0·01	−0·3	8·7	102
15–20	17·7	−0·01	+0·2	7·8	66
20–25	22·2	−0·02	0·0	8·1	48
25–35	29·6	−0·07	+0·3	10·2	51
35–50	41·2	−0·05	+0·1	10·2	22
50+	75·7	+0·01	−0·2	14·1	58

The accordance in position-angle is satisfactory, but in distance there seems to be some systematic error in the first three groups. We may therefore give the first 21 pairs in detail as in Table II. The columns are similar to those in Table I., with the addition of the Oxford and Burnham numbers and epochs, and the measured diameters of the star disks in seconds of arc for each star—instead of the sum of their radii.

TABLE II.

Oxford.	β.G.C.	β. Δ.	$O-\beta$.	β. $\angle$.	$O-\beta$.	Epochs. Oxf.	Epochs. β.	d.	d.
29 68671–0	12453	3·42	−0·45	276·1	−5·1	1895	1902	8·4	7·8
31 64607–8	10949	3·78	−0·57	22·7	−1·8	1902	1902	9·3	7·8
30 15717–6	3399	3·97	−1·27	353·3	+6·8	1903	1902	8·4	6·6
31 60562–3	10388	4·18	−0·01	156·0	+2·6	1902	1902	5·7	5·7
28 42628–9	7656	4·27	−0·76	106·5	+3·1	1901	1902	3·6	3·0
29 38806–7	7656	4·27	−0·58	106·5	−2·4	1906	1902	3·3	3·0
30 17928–9	3897	4·39	−0·49	162·2	−4·2	1903	1886	6·3	6·0
30 18515–6	3897	4·39	−0·07	162·2	+4·7	1903	1886	6·9	6·6
26 70777–6	10865	4·41	−0·63	215·2	+9·3	1897	1901	8·1	2·7
26 39801–0	7764	4·55	−0·13	136·4	+4·8	1899	1901	6·6	6·0
26 1212–0	289	4·84	−0·64	266·1	+4·5	1907	1897	4·2	3·6
26 1578–7	289	4·84	−0·97	266·1	+3·4	1895	1897	7·8	3·6
31 46619–20	9280	5·00	+0·34	68·2	+5·3	1902	1903	6·9	5·4
31 680–1	191	5·03	−0·08	167·5	−1·8	1902	1903	7·5	6·6
28 33047–8	5866	5·21	−1·01	87·4	−1·7	1901	1904	9·6	9·0
29 20785–6	3871	5·31	+0·03	123·8	+2·3	1906	1904	4·2	3·6
31 27401–0	5950	5·41	−0·27	241·7	+7·5	1903	1898	9·6	9·0
28 43968–9	7915	5·58	−0·33	17·6	+6·1	1901	1903	8·1	6·3
28 73437–6	12606	5·80	−0·10	356·9	−6·4	1894	1905	6·3	3·6
29 69412–1	12606	5·80	−1·01	356·9	0·0	1895	1905	7·8	5·4
30 17807–8	3871	5·82	−0·48	125·5	+3·1	1903	1894	6·0	4·5

It may be remarked that four pairs are repeated, being in two zones or on overlapping plates. Attention is also drawn to the fact that there are a number of cases where the epochs differ by more than ten years; that the sum of the radii is, for 15 of the 21 pairs, greater than the Oxford measured distance—indicating that there is no complete separation of the two images on the photographs; that the measures used from Burnham's Catalogue are often on one night by one observer; and that in some stars the visual and photographic magnitudes of the pairs differ relatively, with the result that the primary star used is not the same and the angles differ by 180°: this is, however, unimportant.

Besides the pairs given in Table II. there are 13 pairs—one pair (12 and 13) being in two zones—of an exceptional character; these are shown in Table III., in a similar form to Tables I. and II.

TABLE III.

	Oxford.	β.G.C.	Δ. Oxf.	Δ. β.	∠. Oxf.	∠. β.	Epochs. Oxf.	Epochs. β.	*d.* (in arc). Oxf.	*d.* (in arc). β.
			″	″	°	°			″	″
1	+29 19190–1	3706	1·08	6±	16·0	5±	1907	1820	6·6	5·7
2	+29 63747–8	11353	1·89	3·47	161·8	174·6	1894	1904	2·4	2·1
3	+31 23554–5	5125	2·34	3·58	130·3	144·1	1904	1901	6·6	5·4
4	+30 1508–7	523	2·46	3±	194·1	25±	1902	1820	3·9	3·0
5	+26 10136–4	2064	2·85	8±	297·0	304·7	1893	1831	7·2	3·6
6	+30 66842–1	11122	4·32	6±	262·8	88·0	1895	1828	3·0	3·0
7	+29 14209–8	2905	4·44	5·79	207·7	194·5	1899	1891	14·4	6·6
8	+28 9010–1	2337	4·47	3±	137·3	143·4	1899	1831	7·5	4·2
9	+29 29311–0	5377	4·47	4·81	311·7	331·4	1906	1896	14·7	13·5
10	+26 52969–70	8829	4·71	4·30	54·9	67·6	1897	1901	7·8	4·2
11	+29 15993–4	3147	5·22	1¾	156·9	160±	1896	1820	4·8	3·0
12	+28 47920–1	8570	5·22	3±	12·4	10·8	1901	1828	6·6	6·0
13	+29 45002–3	8570	5·61	3±	15·2	10·8	1900	1828	6·0	4·8

The pairs numbered 2, 3, 7, 9, and 10 show large differences for O – β; no sensible error has been found at Oxford. May these differences be partly due to actual motion? No. 2 (β.G.C., 11353) is somewhat peculiar, and the following words may be quoted from Burnham's Catalogue, p. 962: "There are large errors in some of the distances

1893·28	356°·4	1″·84	2*n*	Ho.
1904·74	175·5	2·37	3*n*	Doo.
1904·87	354·6	3·47	1*n*	Lewis."

The rest of the pairs in this table have Herschel's estimated distances, which are usually several seconds too small; but nothing later is given by Burnham in the catalogue or in the

volume of notes. It seems of little use to reprint so much approximate information, unless connected with some modern accurate measures.

With one or two exceptions the measurement of double stars has been greatly neglected in this country for the last twenty-five or thirty years by possessors of small equatorials of 4 to 8 inches aperture. There must be a number of such instruments available and suitable for double star work that would fall within the power of the smaller equatorials—stars of about 3″ to 30″ distance for example, which are too often avoided or neglected by the more powerful instruments, and perhaps rightly so. A perusal of Burnham's Catalogue would show that there is an abundance of double star work to be done, if only the measurement of those stars with approximate distances or angles be included in the working list.

It seems clear from the results in Table I. that there are systematic errors in *photographic* measures of close pairs: so that, however useful photography may be for wider pairs, it cannot at present replace visual work for close pairs.

The measures from which the Oxford distances and angles have been determined may not be quite as good as could be made by re-measurements and by taking the means in x and y to the fourth decimal (unit 0″·03), but there is danger of prejudice in the bisections, now it is known what is required. The images of the second, and sometimes the third, exposure could also be measured for such an investigation as is briefly given in this paper; the printed measures have, however, only been used now. The formula adopted for these computed angles and distances are (12) and (13) given by Professor H. C. Plummer in his paper (*M.N.*, lxix. p. 102).

To show the distribution of the values of the mean distance residuals (Oxford *minus* Burnham), Tables IV. and V. are given; they correspond to the results shown in Table I., and give the number of pairs with differences of +1″·25 to +0″·75, etc., according to the distances in the heading of each column. Table IV. is with the Oxford distance as the argument, and Table V. is the same for the Burnham distances. There is an apparent change in the visual and photographic distances for the closer pairs.

TABLE IV.

Oxford Distance.

Value of $o-\beta$.	3″.	4″.	5″.	6″.	7″.	8″.	9″–15″.	15″–20″.	20″–25″.	25″–35″.	35″–50″.	50″+.
+1″·25 to +0″·75	0	0	0	0	2	1	10	0	7	3	1	6
+0″·75 to +0″·25	0	0	0	0	6	8	27	9	8	11	3	10
+0″·25 to −0″·25	0	4	3	7	5	7	33	29	20	22	10	28
−0″·25 to −0″·75	5	1	7	9	6	3	27	22	8	16	3	11
−0″·75 to −1″·25	3	1	4	6	2	3	5	3	2	3	2	4
Sum	8	6	14	22	21	22	102	63	45	55	19	59

TABLE V.

Burnham Distance.

Value of $o-\beta$.	3″.	4″.	5″.	6″.	7″.	8″.	9″–15″.	15″–20″.	20″–25″.	25″–35″.	35″–50″.	50″+
+1″·25 to +0″·75	0	0	0	3	0	0	10	3	4	3	1	6
+0″·75 to +0″·25	0	0	0	3	7	6	26	9	8	10	3	10
+0″·25 to −0″·25	0	3	2	10	5	6	33	28	21	22	10	28
−0″·25 to −0″·75	2	4	6	10	4	4	27	21	10	13	6	11
−0″·75 to −1″·25	1	2	2	4	4	3	8	2	3	3	2	4
Sum	3	9	10	30	20	19	104	63	46	51	22	59

1917 *April* 13.

Erratum in Annual Report.

Vol. lxxvii. p. 388, line 6, *for* earlier *read* later.